(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR SCIENCE

COURSE CODE: **STA321/STA 342**

COURSE TITLE: TESTS OF HYPOTHESES

DATE: 17/11/2022 **TIME:** 11:00 AM -1:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define what is meant by

i) Statistical hypothesis

ii) Type I error

iii) Type II error

iv) Critical region

v) Power of the test (5 marks)

b) State and proof the Neyman- Pearson lemma. (10 marks)

c) A latheis adjusted so that the mean of a certain dimension of the parts is 20 cm. A random sample of 10 of the parts produced a mean of 20.3 and standard deviation of 0.2 cm. Do the results indicate that the machine is out of adjustment? Test at the 0.05 level of significance. (6 marks)

d) Based on a single random observation x from the population

$$f(x,\theta) = \begin{cases} \frac{1}{\theta}, & 0 \leq x \leq \theta \\ 0, & otherwise \end{cases}$$

And that you are testing the null hypothesis $H_o : \theta = 1$ versus $H_1 : \theta = 2$by means of a single observed value of x, what would be the sizes of the type one and type two errors if you choose the interval

i) $0.5 \leq x$

ii) $1 \leq x \leq 1.5$ as the critical regions. Also obtain the power function of the test. (9 marks)

QUESTION TWO (20 MARKS)

a) Define what is meant by

i) Most powerful (MP) test

ii) Uniformly most powerful (UMP) test (4 marks)

b) Given the following samples, test the hypothesis, $H_0 : \mu_1 - \mu_0 \leq 3$ versus $H_1 : \mu_1 - \mu_0 > 3$ at 10% level of significance.

Sample 1: 51, 42, 49, 55, 46, 63, 56, 58, 47, 39, 47

Sample 2: 38, 49, 45, 29, 31, 35. (9 marks)

c) State three uses of Chi-square test (3 marks)

d) In investigating several complaints concerning the weight of the jar of a local brand of peanut butter, the Better Business Bureau selected a sample of 36 jars. The sample showed an average net weight of 11.92 ounces and a standard deviation of 0.3 ounce. Using a 0.01level of significance, what would the Bureau conclude about the operation of the local firm?(4 marks)

QUESTION THREE (20 MARKS)

a) Let $(x_1, x_2, \ldots\ldots x_n)$ be a random sample of size n from the normal population with mean μ and variance δ^2, where μ and δ^2 are unknown. Use the likelihood ratio test criteria to obtain the best critical region and test statistic for testing
$H_0 : \mu = \mu_0$ (specified), $0 < \delta^2 < \infty$ against $H_1 : \mu \neq \mu_0$, $0 < \delta^2 < \infty$. (9 marks)

b) Suppose a sample of 50 employees in a particular firm has a mean wage of \$ 160 per week with a standard error of the mean of \$ 1.44. Suppose also that a sample of 40 employees taken from another firm has weekly wage rate of \$ 155 and a standard error of the mean of \$ 1.50. Test the difference between these two means at a 5% level of significance. (5 marks)

c) In anti-malaria campaign in a certain area, quinine was administered to 812 persons out of a total population of 3248. The number of fever cases is as shown below

Treatment	Fever	No fever	Total
Quinine	20	792	812
No quinine	220	2,216	2436
Total	240	3008	3248

Determine whether quinine is useful in checking malaria (take α=5%) (6 marks)

QUESTION FOUR (20 MARKS)

a) Given the eight sample observations 31, 29, 26, 33, 40, 28, 30 and 25. Test at 1% level of significance whether the mean of sample observation is equal to 35. (6 Marks)

b) A coin is tossed 6 times and the hypothesis H_0: $P = \frac{1}{2}$ is rejected if the number of heads is greater than 4. Compute the sizes of Type I and Type II errors if the alternative hypothesis is H_1: $P = \frac{3}{4}$. (6 marks)

c) Prove that $\beta \geq \alpha$ for all 0 in H_1, where α is the significant level and β is the probability of the second kind of error. (8 marks)

QUESTION FIVE (20 MARKS)

a) Explain the following terms

i) Randomized and non-randomized test (2 marks)

ii) Two tailed test (1 mark)

b) A manufacturer of car batteries guarantees that his batteries will last, on the average of 3 years with a standard deviation of 1 year. If 5 of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5 and 4.2 years. Is the manufacturer still convinced that his batteries have a standard deviation of 1 year at$\alpha = 0.05$? (7 marks)

c) The mean score on a widely given freshman mathematics examination is 75. A mathematics teacher at a very large university wants to determine whether there is statistical evidence for claiming that this year's class is not average. Test for this at 5% level of significance using the following scores. (10 marks)

94	69	89	49	88	89	85
95	55	93	86	62	83	96
48	51	69	74	83	71	89
58	89	81	79	52	73	
75	91	68	100	63	81	