(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FORTH YEAR FIRST SEMESTER

SUPPLEMENTARY EXAMINATION

FOR DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

COURSE CODE: **MAP 412**

COURSE TITLE: **MEASURE THEORY**

DATE: 17/11/2022 **TIME:** 8:00 AM - 10:00 AM

INSTRUCTIONS TO CANDIDATES

Answer question ONE and any other two questions

TIME: 2 Hours

QUESTION ONE (20 MARKS)

a) Define the following terms
   i. Ring
   ii. Hereditary $\sigma - ring$
   iii. Outer measure

b) Show that if (μ_j) is ans increasingly directed family of measures on a ring R, and μ is the set function on R defined by the formula $\mu(E) = LUB\mu_j(E)$ then μ is a measure on R

c) Suppose $f_n| \leq |f|\ a.e\ (n = 1,2, \dots)$ where the f_n and g are integrable functions the there exists an integrable function f such that $f = \liminf f_n$ a.e. moreover, $|f| \leq |g| a.e$ and $\int f\ du \leq \liminf \int f_n\ du$

QUESTION TWO (20 MARKS)

a) Define the following terms
   i. Hereditary
   ii. Algebra
   iii. $\sigma - ring$
   iv. v-measure

b) Given that R is a ring of a set Y. show that the monotone class of R, M(R) coincides with the ring generated by R, G(R) ie $M(R) = G(R)$

c) Show that if $\alpha_n \uparrow \alpha$ and $\beta_n \uparrow \beta$ then $\alpha_n + \beta_n \uparrow \alpha + \beta$

QUESTION THREE (20 MARKS)

a) Define the following terms
   i. Lebesque measurable
   ii. Lebesque measure

d) Show that if $f, g \in L^2$ then $|(f|g)| \leq ||f||_2 + ||g||_2$ (Cauchy –schwarz inequality)

e) Show that if μ is measurable on a ring R, and M is the class of all M^*-measurable sets, then $G(R) \subset M$ and the restriction of M^* to G(R) is a measure $\bar{\mu}$ extending μ

QUESTION FOUR (20 MARKS)

a) Show that if f_n is a sequence of integrable functions such that $f_n \geq 0$ a.e and lim inf $\int f_n \, du < \infty$ then there exists an integrable function f such that $f = \liminf f_n$ a.e and one has $\int f \, du \leq \liminf \int f_n \, du$

b)

i. Show that if μ_F is the contradiction of a measure, μ by a fixed set F in a ring R, then μ_F is a measure in R

ii. Show also if $\mu_F < \infty$, then μ_F is a finite measure

QUESTION FIVE (20 MARKS)

a) Show that if $\alpha_n \uparrow \alpha$ and $\beta_n \uparrow \beta$ then $\alpha_n \beta_n \uparrow \alpha\beta$

b) Show that if $f, g \in L^2$ then $||f + g||_2 \leq ||f||_2 + ||g||_2$ (Triangle inequality)

c) Show that if v is an outer measure, then the class M of v −measurable sets is a ring