(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: MAA 426

COURSE TITLE: METHODS II

DATE: 22/11/2022 **TIME 8:00 AM – 10:00 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

1\. (a) State the associativity property and distributivity property of convolution. (2mks)

(b) Consider the equation for a unit circle having. Use Implicit function Theorem to find the formula for the slope of the tangent at any given point (x, y) on the circle. (5mks)

(c) State Gauss's Divergence Theorem. (2mks)

(d) Convert the following equation in terms of polar coordinates.

$$x^2 = \frac{4x}{y} - 3y^2 + 2.$$ (3mks)

(e) State Stokes's Theorem. (2mks)

(f) Prove that $f * g = g * f$ (5mks)

(g) Use the Laplace transforms to solve the IVP,

$$y'' + 2y' + 2y = g(t); y(0) = 1, y'(0) = 1$$ (7mks)

(h) Using commutativity property, find the convolution of $f(t) = e^{-t}$ and $g(t) = \sin(t)$. (4mks)

2\. (a) Compute $L[f(t)]$ where $f(t) = \int_0^t e^{-3(t-\tau)} \cos(2\tau) d\tau$ (5mks)

(b) Find the convolution of $f * g$ given the functions; $f(t) = e^{-kt}; g(t) = e^{at}$ (5mks)

(c) Find the impulse response solution at $t = c$ at the IVP;

$$y_{\&}^{//} + 2y_{\&}^{/} + 2y_{\&} = \delta(t-c) \text{ with } y_{\&}(0) = 0; y_{\&}^{/}(0) = 0; C \in \Re.$$ (10mks)

3\. (a) Let X be an n dimensional Euclidean space. Differentiate between a closed and open ball in X (2mks)

(b) What is a level set of a real valued function of n variables? (2mks)

(c) Describe the nature of the level curves for $f(x, y, z) = z^2 - x^2 - y^2$ (3mks)

(d) Let $g, f; \Re^3 \to \Re$ be continuous function defined as $f(x) = |x|$ and $g(x) = |x|^4$

respectively. If $\int_{\partial B_T} g(t) d\delta = t^2 \int_{\partial B_1} g(t_y) d\delta$ where ∂B_t is a boundary of the ball of radius

$t, y \in B_1$ and $d\delta$ is the area element on the surface of the ball, verify the coarea formula. (13mks)

4. (a) State Green's Theorem. (2mks)

(b) Verify Green's Theorem in a plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region enclosed by; $y = \sqrt{x}$ and $y = x^2$. (18mks)

5. Evaluate the surface integral for $\vec{F} = < xy, yz, zx >$ where S is the surface of triangular prism which occupies x from 0 to 1 and y from 0 to 2, with the base which is flat on xy plane and the top surface is given by plane $z = 1 - x$. (20mks)