



# HYDROLOGICAL DROUGHT FREQUENCY ESTIMATION USING STREAM FLOW DROUGHT INDEX AND MODIFIED GUMBEL METHOD IN UPPER TANA RIVER BASIN

Raphael M. Wambua<sup>1</sup>, Benedict M. Mutua<sup>2</sup>, James M. Raude<sup>3</sup>

<sup>1</sup>Lecturer and PhD Candidate, <sup>2</sup>Associate Professor of Water Resources Engineering, Egerton University Department of Agricultural Engineering, Kenya; <sup>3</sup>Senior lecturer, Sween, Jomo Kenyatta University of Agriculture and Technology, Kenya

## ABSTRACT

**Objective:** To estimate the hydrological drought frequency for upper Tana River basin in Kenya using absolute Stream flow Drought Index (SDI) and modified Gumbel technique. The frequency of drought event of a defined severity for a defined return period is fundamental in planning, designing and operation of water storage systems in the basin.

**Materials and Methods:** Based on a 41-year (1970-2010) stream flow data, hydrological droughts of 2, 5, 10, 20, 50, 100, 200, 500 and 1000-year return periods are evaluated based on the stream flows, Stream flow Drought Index (SDI) and a simplified mathematical model for hydrological drought estimation which is formulated using Gumbel's technique.

**Results:** The absolute SDI increases while the magnitude of the stream flow decreases with return period. The minimum and maximum drought events were exhibited in gauge stations 4AC03 and 4CC03 with absolute SDI ranging from 0.667 to 1.265 and 1.213 to 2.42, and corresponding stream flows of 4.341 to 2.719 and 18.246 to 1.021m<sup>3</sup>/s for a 2 and 1000-year return period respectively.

**Conclusion:** A simplified mathematical model for estimating hydrological drought event that uses mean flows of the annual minimum and average of the first three minimum stream flows as input variables is formulated for different return periods for the river basin.

**Key Words:** Upper tana River basin, SDI, Hydrological drought, Return period, Gumbel technique, Drought frequency, Mathematical model

## INTRODUCTION

Hydrological drought is a natural hazard associated with water deficiency in a hydrological system. Drought may be manifested in below average water availability such as stream flow in rivers, quantity of water in reservoirs, lakes and ground water (Tsakiris, 2009; Mishra and Singh, 2010; Sheffield and Wood 2011). Hydrological drought decreases the availability of water resources (Liu *et al.*, 2012) in river basins, adversely impacting on economic aspects (Carrol *et al.*, 2009; Van Vliet *et al.*, 2012) social dimensions such as increased human conflicts and mortality rates (Garcia-Herrera *et al.*, 2010) and ecological systems (Lake 2011, Lewis *et al.*, 2011). There is need to understand the drought events in order to develop drought mitigation mechanisms in river ba-

sins (Wambua *et al.*, 2014). Hydrological drought impacts on large areas and large human population and may be triggered by climate change and /or variability (Mondal and Mujumdar, 2015). Like other drought events, hydrological drought is considered to be a 'creeping hazard' because it develops slowly, it is not easily noticed, covers extensive areas and it lasts for long a period of time with adverse impact on ecological systems and socio-economic development (Liu *et al.*, 2015; Van-loon, 2015). In addition to hydrological droughts, other types of droughts include meteorological agricultural/soil moisture droughts and socio-economic drought. However, according to Van-loon and Laaha (2015), hydrological drought has the most significant effects almost across different sectors as shown in Table 1.

### Corresponding Author:

Raphael M. Wambua, Lecturer and PhD Candidate, Egerton University Department of Agricultural Engineering, Kenya  
E-mail: wambuarm@gmail.com

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**Table 1: The major drought impacts for different drought categories**

Impact category	Impact sub-category	Hydrological drought	Meteorological drought	Agricultural/ soil moisture drought
Agriculture	Rain-fed	x	x	x
	Irrigation			x
River basins/ecosystems	Terrestrial	x	x	x
	Cooling	x		
Energy and industry	Hydro-power	x		
	Cooling water	x		
Navigation		x		
Drinking water		x		
Recreation		x		

**Source:** Van-loon (2015)

The key parameters of droughts are the longest duration and highest severity for a defined return period. Such parameters aid in designing water storage systems capable of withstanding effects of droughts (Kyambia and Mutua, 2014). Since occurrence of drought contributes to adverse socio-economic impacts, they need to be quantified so as to improvise coping and/or mitigation mechanism. Thus hydrological drought estimation using stream flow data for a defined range of return period in a river basin is crucial. The commonly used return periods are 2, 5, 10, 20, 50, 100, 200, 500 and 1000 years. A 50-year hydrological drought is defined as the drought magnitude which is equalled or exceeded, on average, once per 50 years.

The original version of Gumbel approach in the prediction requires computation of coefficient of variation ( $C_v$ ) and determination of expected mean ( $y_n$ ) and standard deviation ( $\sigma_n$ ) as given in the Gumbel's table. However, in this paper, a mathematical model is formulated for hydrological drought estimation using annual minimum average stream flow and the mean of three lowest stream flows from the recorded data as the main input variables of a modified mathematical model. From an engineering point of view, design of water storage structures requires critical information of longest duration, largest severity for a specific return period. Extreme drought may be treated as a stochastic variable that is challenging to estimate. For practical engineering work, extrapolation and interpolation of drought frequency is crucial for reliable designs. Gumbel (1958) put forth a method for estimating flood frequency. Such a technique may be modified and applied in drought frequency estimation. The Gumbel extreme value distribution for instance has been applied in drought studies in Greece (Dalezios *et al.*, 2000) river basin and Dudhkumar River (Asad *et al.*, 2013).

### Hydrological drought process

The occurrence of hydrological droughts is considered to exhibit stochastic characteristics and thus complex in nature. Hydrological droughts are influenced by hydrological processes of the hydrologic cycle (Peters *et al.*, 2006; Vidal *et al.*, 2010) such as precipitation, evapotranspiration, soil-water storage, runoff flow on land and streams, and groundwater recharge or discharge. The fundamental cause of hydrological drought is climatic change and/or variability. For instance, an abnormally prolonged precipitation deficit leads to low input into hydrological system. Droughts may be triggered by anomalies in temperature in large scale atmospheric and or oceanic patterns and low sea temperature. For any river basin, the rate of depletion of soil-moisture is a function of antecedent moisture condition, evaporation from bare soil surface, evapo-transpiration from vegetated areas, deep percolation of water into the groundwater and runoff into stream networks. For a dry season, runoff and drainage are significantly low while potential evapo-transpiration may be high as a result of increased solar radiation, vapour pressure deficit and wind velocity. During an extreme drought event, soil-moisture may be depleted to wilting point below which plants significantly undergo wilting due to response to the moisture decline in the soil media. This condition leads to reduction in actual evapo-transpiration and locally generated precipitation. The depletion of soil-moisture leads to decrease in recharge of the groundwater storage. Soil-moisture is significantly influenced by the quantity of precipitation, recharge, discharge and aquifer storage and transmissivity characteristics. The relationship between precipitation, soil moisture, runoff, recharge, discharge, ground water and discharge is well explained using the hydrologic water balance Equation 1.

$$R = P - ET - \Delta S - G \quad (1)$$

Where;

$R$ =Runoff (mm)

$P$ =precipitation (mm)

$ET$ =Evapo-transpiration (mm)

$\Delta S$ =Change in soil-water storage (mm)

$G$ = Ground water (mm)

The above relation presents an old concept of hydrology and has been well researched in numerous catchments in the world. However, the application of the hydrologic water balance relationship in drought studies is a relatively new concept (Van loon, 2015). Any climate change and /or variability directly affect precipitation and evapo-transpiration, and indirectly influencing the runoff, soil-moisture storage, and groundwater components of the water balance model. Meteorological droughts propagates into other types of droughts through processes of runoff, stream flow, recharge and discharge which are mainly influenced by river basin characteristics and climatic change or variability. However, the frequency of occurrence of drought is not well researched and thus not understood for numerous river basins in the world.

### Objective

The objective of this research was to estimate hydrological drought frequency using absolute Stream flow Drought In-

dex (SDI) and modified Gumbel's technique for upper Tana River basin, Kenya.

## MATERIALS AND METHODS

### Description of upper Tana River basin

The Tana River basin from which upper Tana River basin is delineated is the largest river basin in Kenya (Jacobs *et al.*, 2004; WRMA, 2010). It lies between latitudes  $00^{\circ} 05'$  and  $01^{\circ} 30'$  south and longitudes  $36^{\circ} 20'$  and  $37^{\circ} 60'$  east. The upper Tana River basin has an area of  $17,420 \text{ km}^2$  (Figure 1). The basin plays a critical role in regulating the hydrology of the entire basin (IFAD, 2012), and in the process, it controls the hydro-electric power generation within the Seven-Folk dams downstream of the Tana River. The basin is very critical in Kenya as it drives the socio-economic development through water supply and agricultural production. The elevation of the upper Tana River basin ranges from approximately 730 m to 4,700 m above mean sea level (a.m.s.l.). These elevations are adjacent to Kindaruma hydro-power dam and Mount Kenya respectively. The dominant soil types in the basin are Andosols, Nitosols, Ferrasols and Vertisols at higher, middle and lower elevations respectively (Jacobs *et al.*, 2004).

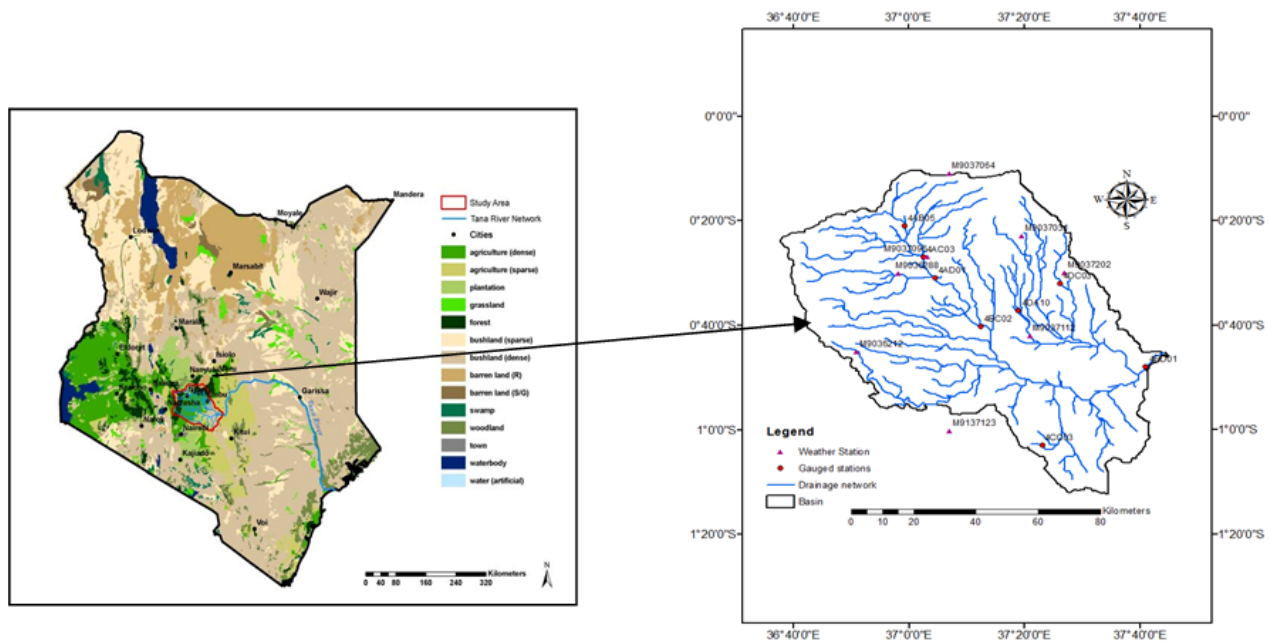


Figure 1: The location of the upper Tana River basin in Kenya

Precipitation and temperature vary spatially across the entire river basin. The annual precipitation at Mount Kenya and the Aberdares ranges is 1800 mm (Otieno and Maingi, 2000). In the mid elevation of 1200 to 1800 m a.m.s.l., the annual rainfall ranges from 1000 to 1800 mm, while the lower elevations at 1000 m, and receive annual rainfall of 700 mm. The basin is characterized by seasonal rainfall fluctuations as influenced by orographic forces (Saenyi, 2002). Subsequently, this leads to seasonal variation of stream flows in Tana River. Generally the basin experiences bimodal rainfall pattern which is triggered by inter-tropical convergence zone (Wilschut, 2010). The two main rain seasons as shown in Figure 2, are distributed in the months of March to June, and September to December where the monthly average precipitation is considerably high compared to the other months.

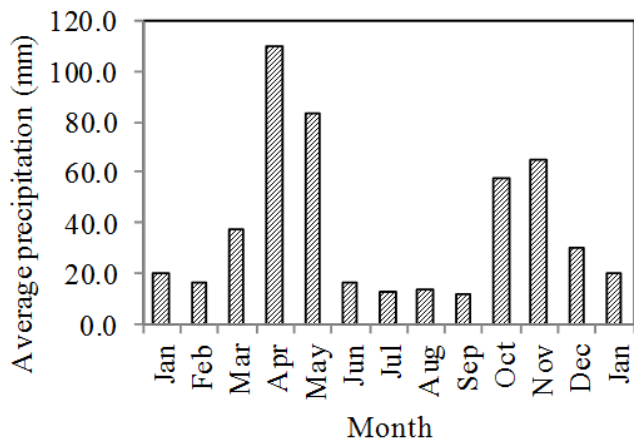


Figure 2: The monthly average precipitation

**Data acquisition**

Stream flow data used in the present study was obtained from the Ministry of Water and Irrigation, and Water Resources Management Authority (WRMA) for eight stations for a period of 41 years (1970-2010). Data from eight stations with consistent data that had less than 20% missing data was selected for the study. The Double mass curve was used to check for the data consistence.

**Gumbel’s extreme value (EV1) method**

Gumbel’s method was originally developed for flood estimation. However, it has previously been adopted in drought studies (Dalezios et al., 2000). The form of Gumbel technique used for the present study for estimating extreme drought event is expressed as:

$$Q_T = \bar{Q}(1 + KC_v) \tag{2}$$

Where;

$Q_T$ =the probable hydrological drought discharge with a return period of T years

$C_v$ =coefficient of variation

$\bar{Q}$ =The mean hydrological drought discharge (m<sup>3</sup>/s)

$K$ = frequency factor

The frequency factor and the coefficient of variation are determined from the relation:

$$K = \frac{(y_T - y_n)}{\sigma_n} \tag{3}$$

$$C_v = \frac{\sigma}{\bar{Q}} \tag{4}$$

$$y_T = -\ln \ln \left( \frac{T}{T-1} \right) \tag{5}$$

Where;

$y_n$ , =expected mean

$\sigma_n$  =standard deviation of reduced drought extremes estimated from Gumbel’s Table

In an attempt to simplify Equation (5), Powell (Asad et al., 2013) developed an equation to estimate K using the relation:

$$K = \left[ \frac{(\sqrt{6}) \times (0.5772 + \ln \ln T / T - 1)}{\pi} \right] \tag{6}$$

Although the above function improved the method of estimating the frequency factor K, which can now be computed based on return period T and not number of years of record, the method for calculating Cv still remains as suggested by Gumbel.

**Stream flow drought index**

A drought index is an integration of either one or more of hydro-meteorological variables such as precipitation, stream flow, soil moisture, temperature, ground water, water reservoir volume or level (Sun et al., 2011). In this study, Stream flow drought index (SDI) that uses stream flow data is applied. The SDI for each gauged station was determined using the following relation:

$$SDI_i = \frac{(Q_i - \bar{Q}_k)}{\sigma_k} \tag{7}$$

Where;

$SDI_i$ =stream flow drought index for  $i^{th}$  hydrological month

$Q_i$ =stream flow for the  $i^{th}$  hydrological month

$K$ =length of period of data record/reference period

$\sigma_k$ =the standard deviation of the cumulative stream flow volumes for  $k^{th}$  reference period

**Table 2: Definition of states of drought based on SDI**

State	Drought description	Criterion
0	Non drought	$SDI \geq 0.0$
1	Mild drought	$-1.0 \leq SDI < 0.0$
2	Moderate drought	$-1.5 \leq SDI < -1.0$
3	Severity drought	$SDI < -2.0$
4	Extreme drought	$SDI < -2.0$

The original function was developed by Gumbel (Gumbel, 1958) for extreme flood estimation that used data that exhibit positive values. In this research, the stream flow drought index with negative values as shown in Table 2 represent the period of drought episodes. These negative values are converted to their corresponding absolute values and fitted to Equation (2). Then the  $SDI$  for 2, 5, 10, 20, 50, 100, 200, 500 and 1000-year return periods were computed using Equation (7). The resulting  $SDI$  data was arranged into ascending order alongside the corresponding stream flow. The stream flow corresponding to the computed  $SDI_m$  of rank  $m$  for specific return period was selected. Those stream flow values without corresponding  $SDI$  were interpolated using the relations:

$$Q_m = Q_o + \frac{Q_1 - Q_o}{SDI_1 - SDI_o} \times (SDI_m - SDI_o) \quad (8)$$

Where;

$Q_m$  = the stream flow of rank  $m$  and specific return period ( $m^3/s$ )

$Q_1$ =higher rank stream flow ( $m^3/s$ )

$Q_o$ =lower rank stream flow ( $m^3/s$ )

$SDI_1$ =the higher stream flow drought index

$SDI_o$  = the lower rank stream flow drought index

$SDI_m$  = interpolated value of stream flow drought index

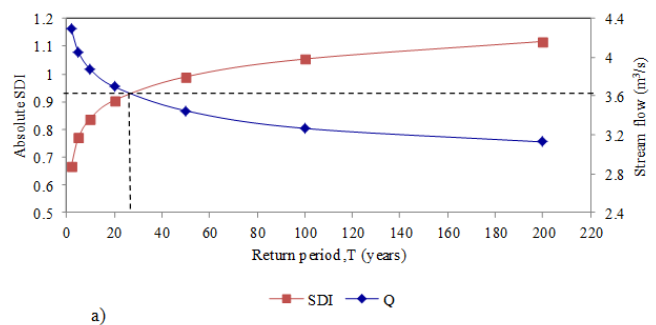
**Table 3. Absolute stream flow drought index (SDI) for computation of  $Q_m$**

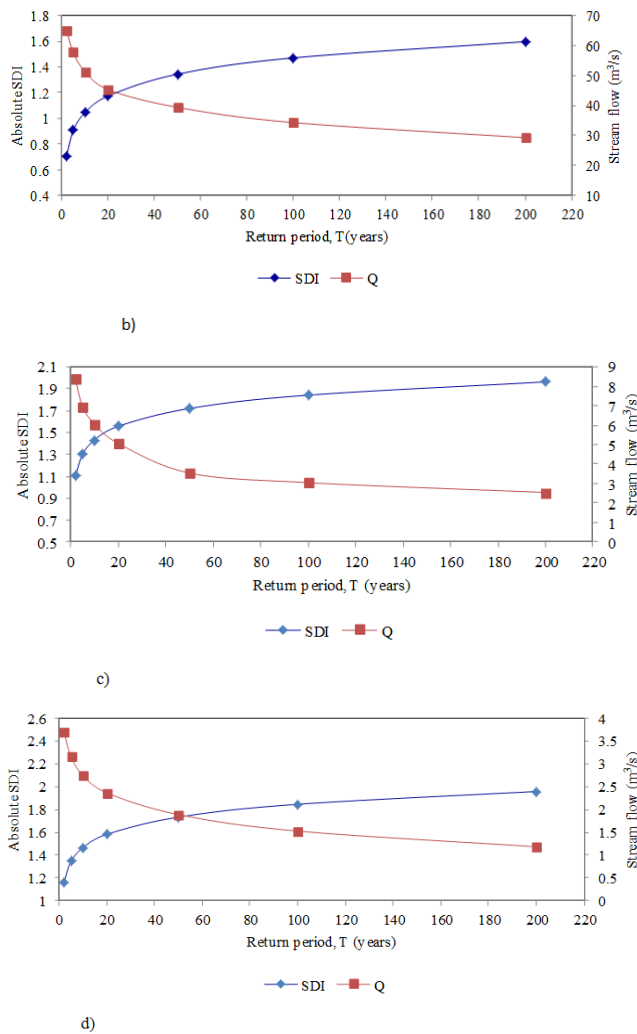
Absolute SDI	Stream flow $Q_m$ ( $m^3/s$ )
0.4609	4.8339
0.4809	4.7814
0.5229	4.6708
0.5692	4.5494
0.5753	4.5334
0.5797	4.5217
0.5873	4.5018
0.6119	4.4371
0.6485	4.3408
0.6790	4.2610
0.7226	4.1463
0.7246	4.14098

## RESULTS AND DISCUSSION

For instance for a 2-year return period, whose computed  $SDI_m$  is 0.6665,  $Q_m$  value was interpolated using Equation (8) based on the data in Table 3.

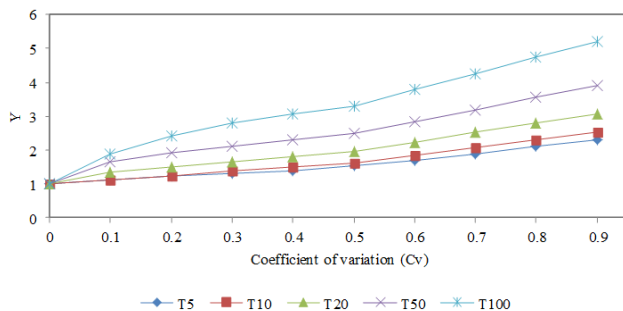
The results of the fitted curves show that the absolute  $SDI$  increases with the return period in all gauged stations (4AB05, 4BC02, 4AC03 and 4AD01) as given in Figure 3. For instance, hydrological droughts represented by magnitude of absolute  $SDI$  of 0.667 and 1.265 are equaled or exceeded once on average every 2 and 1000 years respectively. The same applies to the other hydrological droughts of defined absolute  $SDI$ . For water resources managers, data on stream flow is important for ease of water resources planning and management. Thus in this research, the absolute  $SDI$ s are tied to their respective stream flow magnitudes.





**Figure 3:** The relationship between the  $Q_m$ ,  $SDI$  and return period for (a) 4AB05 (Amboni), b) 4BC02 (Tana Sagana) c) 4AC03 (Sagana) and (d) 4AD01 (Gura) gauge stations

Generally the results show that, the minimum and maximum drought episodes were exhibited in gauge stations 4AB05 (Amboni) and 4CC03 (Yatta furrow) with absolute  $SDI$  ranging from 0.667 to 1.265 and 1.213 to 2.42 for 2 and 1000-year return period respectively.



**Figure 4:**  $Q_T / \bar{Q}$  versus  $C_v$  for 4ED01 (Kamburu) gauge station in upper Tana River basin

Using Table 2 and results from Figure 3, the critical points are identified. The critical point is the level of hydrological drought beyond which the water facilities is significantly affected by drought. For instance the critical point for 4AB05 (Amboni) gauge station, as shown by dotted line, coincides with return period of 28 years with absolute  $SDI$  of 0.92 and stream flow of  $3.6 \text{ m}^3/\text{s}$  (Figure 3a), while that of 4BC02 (Tana sagana) is 20 years with absolute  $SDI$  of 1.2 and stream flow of  $45 \text{ m}^3/\text{s}$  (Figure 3b). In this case, if a water resource system is to be designed for example at the gauging stations 4AB05 and 4BC02, the systems should be designed for return periods of less or equal to 28 and 20 years respectively.

Form Figure 4, the results show that the ratio of  $Q_T / \bar{Q}$  represented by  $Y$  increases with  $C_v$  for different return periods. This confirms that the Gumbel method is also applicable in drought frequency estimation just like in flood frequency analysis (Al-Mashindani *et al.*, 1978).

### Development of the modified mathematical model

Gumbel method has been modified before for flood studies. However, scanty research as far as its application in drought studies is concerned. In this research the principles used by Al-Mashindani (1978) in flood assessment was used in hydrological drought estimation for upper Tana River basin. The value of  $Q_T$  for drought studies in Gumbel's technique is written as:

$$Q_T = \bar{Q} \left[ 1 + \frac{c_v (y_T - y_n)}{\sigma_n} \right] \quad (9)$$

Considering a stream flow drought index ( $SDI_m$ ) with a rank  $m$  that corresponds to a particular stream flow  $Q_m$ , then by applying Equation (9) this results to:

$$Q_m = \bar{Q} \left[ 1 + \frac{c_v (y_m - y_n)}{\sigma_n} \right] \quad (10)$$

When Equations (9) and (10) are reorganized and then dividing Equation (9) by (10) the relation becomes:

$$\frac{Q_T - \bar{Q}}{Q_m - \bar{Q}} = \frac{y_T - y_n}{y_m - y_n} \quad (11)$$

Simplifying Equation (11) leads to:

$$\frac{Q_T - \bar{Q}}{Q_m - \bar{Q}} = \frac{y_T - y_n}{y_m - y_n} \quad (12)$$

From Gumbel's method the estimated values of  $y_n$  as given in Table 5 are 0.5236 and 0.5745 for the data record of 20 and 1000 years respectively. Thus the parameter  $y_n$  can be assumed to be a constant that arbitrarily lies between 0.5236 and 0.5747 and estimated using the relation:

**Table 4. Variation of absolute SDI and the stream flows for defined return periods**

Stream Name ID	Gauge ID	Q <sub>2</sub>		Q <sub>5</sub>		Q <sub>10</sub>		Q <sub>20</sub>		Q <sub>50</sub>		Q <sub>100</sub>		Q <sub>200</sub>		Q <sub>500</sub>		Q <sub>1000</sub>			
		SDI	Q	SDI	Q	SDI	Q	SDI	Q	SDI	Q	SDI	Q	SDI	Q	SDI	Q	SDI	Q	SDI	Q
Amboni	4AB05	0.666	4.341	0.770	4.006	0.839	3.921	0.905	3.687	0.989	3.449	1.054	3.268	1.117	3.128	1.201	2.902	1.265	2.719		
Tana-sagana	4BC02	0.701	65.242	0.908	57.734	1.044	51.343	1.175	45.324	1.345	39.403	1.472	34.325	1.598	29.283	1.765	22.285	1.891	16.988		
Sagana	4AC03	1.102	8.3549	1.301	6.9633	1.431	6.059	1.559	5.071	1.792	3.543	1.841	3.045	1.963	2.542	2.124	1.239	2.245	0.404		
Gura	4AD01	1.159	3.706	1.343	3.152	1.465	2.743	1.581	2.363	1.732	1.887	1.845	1.525	1.958	1.179	2.206	0.703	2.218	0.453		
Yatta- furrow	4CC03	1.213	18.246	1.468	15.318	1.585	12.987	1.691	11.851	1.864	9.013	1.932	7.122	2.022	4.840	2.219	2.234	2.42	1.021		
Nyamindi	4DA10	1.124	16.562	1.411	15.283	1.472	13.971	1.625	11.106	1.809	9.102	1.985	7.024	2.011	4.901	2.212	2.258	2.339	1.025		
Rupingazi	4DC03	1.113	3.846	1.307	3.219	1.436	2.841	1.559	2.434	1.719	1.936	1.839	1.554	1.958	1.178	2.116	0.651	2.235	0.276		
Kamburu	4ED01	1.112	19.463	1.319	16.182	1.453	14.071	1.592	12.044	1.764	9.332	1.893	7.382	2.022	5.401	2.192	2.447	2.342	1.114		

$$y_n = 0.5236 + \frac{1}{2} \times (0.5747 - 0.5236) = 0.54915 \approx 0.5 \quad (13)$$

The computed value of 0.55 corresponds to hydrological drought event of a particular severity with a return period of 50 years (Table 5).

**Table 5: Typical values of  $y_n$  and  $\sigma_n$  in Gumbel's method**

N	$y_n$	$\sigma_n$	N	$y_n$	$\sigma_n$
20	0.5236	1.0628	70	0.5548	1.1824
25	0.5309	1.0915	75	0.5559	1.1898
30	0.5362	1.1124	80	0.5569	1.1938
35	0.5403	1.1285	85	0.5578	1.1973
40	0.5436	1.1413	90	0.5589	1.2007
45	0.5463	1.1518	95	0.5593	1.2038
50	0.5465	1.1607	100	0.5600	1.2065
55	0.5504	1.1681	200	0.5672	1.2359
60	0.5521	1.1747	500	0.5724	1.2588
65	0.5536	1.1808	1000	0.5745	1.2685

N=number of years,  $y_n$ =expected mean of the data,  $\sigma_n$ =expected standard deviation of the data

Source: Gumbel (1958)

Based on Gumbel's method the return period  $T$  and  $y_T$  are determined using the following functions:

$$T = \frac{N+1}{m} \quad (14)$$

$$y_T = -\ln \ln \left( \frac{T}{T-1} \right) \quad (15)$$

For a particular stream flow drought index corresponding to stream flow  $Q_m$  with a rank  $m$ , the value of  $y_m$  is determined from:

$$y_m = -\ln \ln \left[ \left( \frac{N+1}{m} \right) + \left( \frac{N+1}{m} - 1 \right) \right] = -\ln \ln \left[ \left( \frac{N+1}{m} \right) \times \left( \frac{m}{N+1-m} \right) \right] \quad (16)$$

This relation reduces to:

$$y_m = -\ln \ln \left( \frac{N+1}{N+1-m} \right) \quad (17)$$

Substituting Equation (15) into Equation (10) results to:

$$\frac{Q_T - \bar{Q}}{Q_m - \bar{Q}} = \frac{-\ln \ln \left( \frac{T}{T-1} \right) - 0.55}{-\ln \ln \left( \frac{N+1}{N+1-m} \right) - 0.55} \quad (18)$$

$$\frac{Q_T - \bar{Q}}{\frac{1}{3} \sum_{m=1}^3 Q_m - \bar{Q}} = \frac{-\ln \ln \left( \frac{T}{T-1} \right) - 0.55}{-\frac{1}{3} \sum_{m=1}^3 \ln \ln \left( \frac{N+1}{N+1-m} \right) - 0.55} \quad (19)$$

From the principles of Schulz (1973), it can be shown that:

$$-\ln \ln \left( \frac{T}{T-1} \right) = \ln T - \left( \frac{1}{2T} + \frac{1}{24T^2} + \frac{1}{8T^3} \right) \quad (20)$$

Neglecting the quantities inside the brackets on the right side of Equation (20), it reduces to:

$$-\ln \ln \left( \frac{T}{T-1} \right) = \ln T \quad (21)$$

Also neglecting the quantities in the brackets for Equation (17) yields:

$$y_m = -\ln \ln \left( \frac{N+1}{N+1-m} \right) = \ln \left( \frac{N+1}{m} \right) \quad (22)$$

Therefore, Equation (19) can be reduced further into the following relation:

$$\frac{Q_T - \bar{Q}}{\frac{1}{3} \sum_{m=1}^3 Q_m - \bar{Q}} = \frac{\ln T - 0.55}{-\frac{1}{3} \sum_{m=1}^3 \ln \left( \frac{N+1}{m} \right) - 0.55} \quad (23)$$

Table 4 show stream flow values corresponding to absolute stream flow drought index that were computed for different return periods using Equations (22) and (23) based on data acquired for the upper Tana River basin.

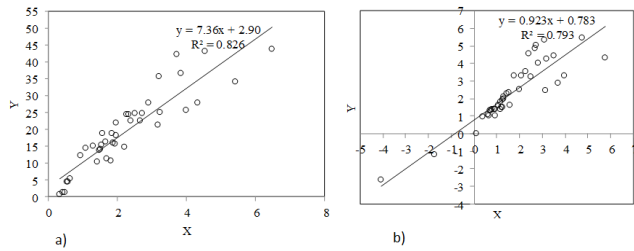
By redefining Equation (23) in X and Y as shown in Equations (24) and (25), and plotting the corresponding data for the upper Tana River basin gives Figure 5.

$$X = \frac{\ln T - 0.55}{-\frac{1}{3} \ln \left( \frac{N+1}{m} \right) - 0.55} \quad (24)$$

$$Y = \frac{\bar{Q} - Q_T}{\bar{Q} - \frac{1}{3} \sum_{m=1}^3 Q_m} \quad (25)$$

The results from Figure 5 show that all the gauged stations exhibit strong linear correlation.





**Figure 5:** Fitted Data for 4BC02 (Tana Sagana) and 4AC03 (Sagana) gauged stations

The plot shows that there is a strong correlation between the expressions on the left and right side of Equation (23), with correlation coefficients at gauge stations of IDs 4BC02 (Tana Sagana) and 4AC03 (Sagana) of 0.826 and 0.793 respectively. This means that the stream flow  $Q_T$  of any return period can be determined from mean flows of the annual minimum and the average of the first three minimum mean stream flows for the upper Tana river basin as per Equation (23). This is found to be consistency with the similar plot developed for flood estimation by Al-Mashindani *et al.* (1978).

## CONCLUSION

- i) From the study SDI, stream flows have been explored and their corresponding return periods estimated. It is concluded that the computed absolute SDI vary across the gauge stations and increase while the corresponding stream flow decline with increase in return period
- ii) A simplified mathematical model for estimating hydrological drought event that uses the mean of the annual minimum and average of the first three minimum stream flows as input variables is developed for different return periods in the upper Tana River basin.
- iii) Critical points upon which design of water storage systems can be based are identified for different gauge stations. These indicate the level of hydrological drought beyond which the water facilities is significantly affected by the drought.

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