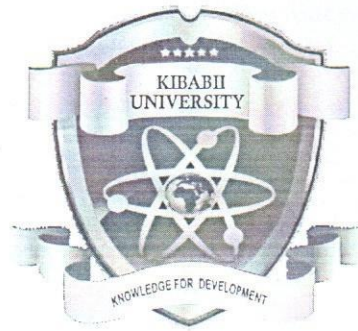


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(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR
FOURTH YEAR SECOND SEMESTER
SPECIAL/SUPPLEMENTARY EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN
MATHEMATICS

COURSE CODE: MAP 421

COURSE TITLE: TOPOLOGY II

DATE: 22/11/22

TIME: 11.00 AM -1.00 PM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

- a) Define the following terms
- (i). Hausdorff space X . (2 mks)
 - (ii). Unit ball in \mathbb{R}^n (2 mks)
 - (iii). First countable space (2 mks)
 - (iv). Normal space (2 mks)
 - (v). T_1 space (2 mks)
- b) What do you understand by the term a complete regular space? Give an example. (4 mks)
- c) Let X and Y be topological spaces and $f: X \rightarrow Y$ a continuous function. Show that its image is compact if X is compact. (6 mks)
- d) (i). Show that the subspace $Y = [0,1]$ of a real line is connected. (4 mks)
(ii). Give an example of a subset of Y that is not connected (1 mk)
- e) Prove that every subspace of a second countable space is second countable (5 mks)

QUESTION TWO (20 MARKS)

- a) What is a linear continuum? (2 mks)
- b) Let $I \times I$ be a product topological space and π_1, π_2 be projections on I respectively be defined as $\pi_1(x, y) = x$ and $\pi_2(x, y) = y$ for $x, y \in I$. Let $A \subset I \times I$ be square $A = \{x, y: a \leq x \leq b, c \leq y \leq d, a, b, c, d \in \mathbb{R}\}$. Show that A is a linear continuum. (8 mks)
- c) State and prove the generalization of extreme value theorem. (10 mks)

QUESTION THREE (20 MKS)

- a) Define a T_2 space given an example. (3 mks)
- b) What is path connected space? Give an example (3 mks)
- c) Every metrizable space is normal. (9 mks)
- d) Define the term a compact space, hence show that space of real numbers, \mathbb{R} , is not compact. (5 mks)

QUESTION FOUR (20 MARKS)

- a) Show that the interval $B = (0,1)$ of the real line with the usual topology is not sequentially compact. (5 mks)

- b) Let $\{A_i\}_{i \in I}$ be a collection of connected subspaces with a common point. Show that $\bigcup_{i \in I} A_i$ is connected. (5 mks)
- c) Prove that every open covering of a space X with a countable basis contains a countable sub collection covering X . (6 mks)
- d) Find the smallest compact set A containing (p, q) given that $p, q \in \mathbb{R}$. Can there be a separation on A ? Explain? (4 mks)

QUESTION FIVE (20 MARKS)

- a) Show that the space \mathbb{R}_l is normal. (5 mks)
- b) Let X be a topological space. Define a relation $x \sim y$ on X if there is a connected subspace of X containing both x and y . Show that \sim is an equivalence relation. (6 mks)
- c) When is a collection of subsets of a space X said to have a finite intersection property? (2 mks)
- d) Define a set X as $X = \{e, f\}$. For a T_1 topological space from the set. Show that space formed is a topological spaces but not T_1 . (7 mks)