(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

COURSE CODE: **MAP 312**

COURSE TITLE: LINEAR ALGEBRA III

DATE: 22/11/2022 **TIME:** 11 AM -1 PM

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Define the following: (3 marks)
   (i) A quadratic function on V.

a) Show that if matrix A is Hermitian, then all the eigenvalues of A are real. (10 marks)

b) Show that matrix A= $\begin{bmatrix} 5 & -3 & 2 \\ 15 & -9 & 6 \\ 10 & -6 & 4 \end{bmatrix}$ is a nilpotent matrix of index 2. (4 marks)

c) Outline any five applications of finite vector spaces. (5 marks)

d) Given that v = (1 +2i, 3-i) and u = (-2 +i, 4) are vectors in the complex vector space C^2, determine the vector 3v-(5-i) u. (4 marks)

e) Show if the matrix A = $\begin{bmatrix} 2 & -1 \\ 1 & -2 \end{bmatrix}$ of index 2 is nilpotent or not. (4 marks)

QUESTION TWO (20 MARKS)

a) Define the terms: (2 marks)
   (i) Hermitian matrix. (2 marks)
   (ii) Symmetric matrix. (2 marks)
   (iii) Nilpotent matrix.

b) Show that S = $\{(i,0,0) \quad ,(i,i,0) \quad ,(0,0,i)\}$
   where $v_{1=}\ (i, \quad 0, \quad 0)$
   $v_{2=}\ (i, \quad i, \quad 0)$
   $v_{3=}\ (0, \quad 0, \quad i)$
   is a basis for C^3. (10 marks)

c) Calculate the conjugate transpose of matrix A.

A = $\begin{bmatrix} 1 & -2-i & 5 \\ 1+i & i & 4-2i \end{bmatrix}$ (4 marks)

QUESTION THREE (20 MARKS)

a) Define the terms:

(i) Jordan block. (2 marks)

(ii) Jordan form. (2 marks)

(iii) Jordan chain. (2 marks)

b) Determine the Jordan form of the operator represented by the matrix

$$A=\begin{pmatrix} 1 & -1 & 0 \\ -1 & 4 & -1 \\ -4 & 13 & -3 \end{pmatrix}$$ (7 marks)

c) Show that if (V, q) is a quadratic form over $F = F_2$ and that dim $V \geq 4$, there exists a Vector $v \in V$ with $q(v) = 0$. (7 marks)

QUESTION FOUR (20 MARKS)

a) Define the following:

(i) Eigenvalue and eigenvector. (3 marks)

(ii) Determinant of a matrix. (2 marks)

(iii) Trace of a matrix. (2 marks)

b) Given the following 2×2 matrix $A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$, find its eigen equation. (10 marks)

c) Given that A is a real matrix with a complex eigenvalue $\lambda = \mu + i\nu$ and corresponding eigenvector $\nu = x + iy$, find the complex conjugate λ and the complex conjugate eigenvector. (3 marks)

QUESTION FIVE (20 MARKS)

a) Define the following:

(i) Bilinear form over F. (2 marks)

(ii) B-Orthogonal compliment. (2 marks)

(iii) An isometry. (4 marks)

(iv) Tensor product. (2 marks)

b) Show that if matrix A is Hermitian, then any two eigenvectors from different eigenspaces are orthogonal in the standard inner product for $\mathbb{C}^n$, ($\mathbb{R}^n$, If A is real symmetric). (10 marks)