(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: STA 311/STA 341

COURSE TITLE: THEORY OF ESTIMATION

DATE: 17/11/2022 TIME: 11:00 AM – 1:00 PM

INSTRUCTIONS TO CANDIDATES
Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the terms below as used in statistics. (3 marks)
   (i) Parameter
   (ii) Statistic
   (iii) An estimator

b) State any three requirements of a good estimator. (3marks)
c) Proof that the sample mean is an unbiased estimator of population mean. (4marks)
d) Differentiate between parametric and non- parametric statistical test in statistics giving relevant examples. (4 marks)
e) Let $x1, \ldots, xn$ be random sample from a poison distribution with parameter λ. Find the estimator for λ, using any suitable method. (5 marks)
f) Proof that the sample mean square S^2 is an unbiased estimator of the population variance. (4 marks)
g) Explain the concept of MVUE in estimation. (4marks)
h) Explain the meaning of the following terms: (3 marks)
(i) Interval estimation
(ii) Point estimation
(iii) Confidence interval

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QUESTION TWO (20 MARKS)

a) Discuss the concept of sufficiency as used in estimation theory (5 marks)
b) Explain the Cramer-Rao inequality (6 marks)
c) let $x_1, \ldots, x_n$ be a random sample from a Poisson distribution. Show that $T = Max\ (x_1, \ldots, x_n)$ is a sufficient statistic for λ. (9 marks)

QUESTION THREE (20 MARKS)

a) Let $x_1, \ldots, x_n$ be a random sample from a normal distribution . Find the MLE of μ *and* δ^2 (10 marks)
b) Let $x_1, \ldots, x_n$ be a random sample from a uniform distribution in $(\theta - \frac{1}{2}, \theta + \frac{1}{2})$. Find the MLE of θ (10 marks)

QUESTION FOUR (20 MARKS)

4. (a) i. When is an estimator t_n of θ said to be sufficient? (2 mks)

ii. For a normal population $N(\mu, \sigma^2)$ if σ^2 is known, prove that $\bar{X}$ is sufficient estimator for μ (5 mks)

iii. If μ is known, prove that s^2 is not a sufficient estimator for σ^2 and hence find its sufficient estimator. (3 mks)

(b) X is a binomial random variable with parameters n (known) and P (unknown). Given a random sample of N observations of X (10 mks)

i. Compute the method of moments estimator for p

ii. What will be the method of moments estimator for n and p when both are unknown

QUESTION FIVE(20 MARKS)

5. (a) A random sample is taken from a normal population with mean 0 and variance σ^2. Examine if $s^2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2$ is MVUE for σ^2 (8 mks)

(b) A random sample is picked from a population whose distribution is given by;

$$f(x; \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$$

for $x = 0, 1, 2, \ldots, \infty$

Taking $\Psi(\lambda) = e^{-\lambda}$, find Cramer-Rao lower bound for t, where t is unbiased estimator for $\Psi(\lambda)$ (7 mks)

(c) Let $x_1, ..., x_n$ be iid random variable from a Bernoulli distribution with parameter P. Show that $T = \sum_i^n X_i$ is a sufficient statistic (5 mks)