



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR
THIRD YEAR FIRST SEMESTER
SPECIAL/SUPPLEMENTARY EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION AND
BACHELOR OF SCIENCE

COURSE CODE: STA 311/STA 341

COURSE TITLE: THEORY OF ESTIMATION

DATE: 17/11/2022

TIME: 11:00 AM – 1:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **3** Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

- a) Define the terms below as used in statistics. (3 marks)
- (i) Parameter
 - (ii) Statistic
 - (iii) An estimator
- b) State any three requirements of a good estimator. (3marks)
- c) Proof that the sample mean is an unbiased estimator of population mean. (4marks)
- d) Differentiate between parametric and non- parametric statistical test in statistics giving relevant examples. (4 marks)
- e) Let x_1, \dots, x_n be random sample from a poisson distribution with parameter λ . Find the estimator for λ , using any suitable method. (5 marks)
- f) Proof that the sample mean square S^2 is an unbiased estimator of the population variance. (4 marks)
- g) Explain the concept of MVUE in estimation. (4marks)
- h) Explain the meaning of the following terms: (3 marks)
- (i) Interval estimation
 - (ii) Point estimation
 - (iii) Confidence interval

QUESTION TWO (20 MARKS)

- a) Discuss the concept of sufficiency as used in estimation theory (5 marks)
- b) Explain the Cramer-Rao inequality (6 marks)
- c) let x_1, \dots, x_n be a random sample from a Poisson distribution. Show that $T = \text{Max}(x_1, \dots, x_n)$ is a sufficient statistic for λ . (9 marks)

QUESTION THREE (20 MARKS)

- a) Let x_1, \dots, x_n be a random sample from a normal distribution . Find the MLE of μ and δ^2 (10 marks)
- b) Let x_1, \dots, x_n be a random sample from a uniform distribution in $(\theta - \frac{1}{2}, \theta + \frac{1}{2})$. Find the MLE of θ (10 marks)

QUESTION FOUR (20 MARKS)

4. (a) i. When is an estimator t_n of θ said to be sufficient? (2 mks)
ii. For a normal population $N(\mu, \sigma^2)$ if σ^2 is known, prove that \bar{X} is sufficient estimator for μ (5 mks)
iii. If μ is known, prove that s^2 is not a sufficient estimator for σ^2 and hence find its sufficient estimator. (3 mks)
- (b) X is a binomial random variable with parameters n (known) and P (unknown). Given a random sample of N observations of X (10 mks)
- i. Compute the method of moments estimator for p
ii. What will be the method of moments estimator for n and p when both are unknown

QUESTION FIVE(20 MARKS)

5. (a) A random sample is taken from a normal population with mean 0 and variance σ^2 . Examine if $s^2 = \frac{1}{n} \sum_{i=1}^n X_i^2$ is MVUE for σ^2 (8 mks)
- (b) A random sample is picked from a population whose distribution is given by;

$$f(x; \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}$$

for $x = 0, 1, 2, \dots, \infty$

Taking $\Psi(\lambda) = e^{-\lambda}$, find Cramer-Rao lower bound for t , where t is unbiased estimator for $\Psi(\lambda)$ (7 mks)

- (c) Let x_1, \dots, x_n be iid random variable from a Bernoulli distribution with parameter P . Show that $T = \sum_{i=1}^n X_i$ is a sufficient statistic (5 mks)