(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUATION AND BACHELOR OF SCIENCE

COURSE CODE: MAP 112

COURSE TITLE: BASIC MATHEMATICS AND ANALYTIC GEOMETRY

DATE: 23/12/2022   **TIME: 2:00 PM – 4:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms
   i.   Orthogonal circles
   ii. Hyperbola
   iii. Ellipse   (3 marks)

b) Show that two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ where $b_1, b_2 \neq 0$ are
   i.   Parallel if $\frac{a_1}{b_1} = \frac{a_2}{b_2}$
   ii. Perpendicular if $a_1a_2 + b_1b_2 = 0$ (3 marks)

c) Solve the following equation using synthetic division tableau
   $(x^3 + 4x^2 - 5x - 14) \div (x - 2)$ (3 marks)

d) Convert the following parametric equation to Cartesian form, analyze and sketch its graph
   $x = -4 + 2\cos\theta, \; y = 1 + \sin\theta, \; 0 \leq \theta \leq 2\pi$ (8 marks)

e) Find equations of lines passing through $(-3, 8)$ and cut off intercepts from coordinate
   axes such that their sum is 7.   (5 marks)

f) Determine whether the lines $3x + 4y = 9$ and $6x + 8y = 15$ are parallel or not. (3 marks)

g) Analyze and sketch the graph of $y^2 + 4y - 4x + 3 = 0$ (5 marks)

QUESTION TWO (20 MARKS)

a) Sketch the graphs of the following equations indicating their respective equations
   features.
   i. $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

   ii. $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ (6 marks)

b) Analyze $9x^2 - 4y^2 + 54x + 8y + 78 = 0$. Sketch its graph.   (9 marks)

c) Find the directrix, eccentricity and sketch the graph of $\; r = \dfrac{4}{2 + \cos\theta}$ (5 marks)

QUESTION THREE (20 MARKS)

a) Convert the following equation to polar $(x - a)^2 + (y - b)^2 = a^2 + b^2$ (4 marks)

b) Find the equation of a parabola, directrix and its sketch having its vertex at $(-1, 2)$ and focus at $(0,2)$.   (5 marks)

c) Convert the Cartesian equations given below to parametric equations.

    i. $\frac{(x-2)^2}{64} + \frac{(y+1)^2}{4} = 1$ (3 marks)

    ii. $\frac{x^2}{9} - \frac{y^2}{4} = 1$ (2 marks)

    iii. $y^2 = 8x$ (2 marks)

d) Convert the following polar coordinates to rectangular coordinates

    i. $\left(-2, \frac{\pi}{4}\right)$

    ii. $\left(-1, \frac{2\pi}{3}\right)$ (4 marks)

QUESTION FOUR (20 MARKS

a) Find equation of an ellipse with foci $(0, -3)$ and focus at $(0,3)$ that passes through $(0,5)$ (4 marks)

b) Find the zeros of $f(x) = x^3 - 6x^2 + 11x - 6$ (3 marks)

c) Analyze and sketch the graphs of

    i. $12x^2 + 20y^2 - 12x + 40y = 37$ (7 marks)

    ii. $4y^2 - 8y + 3x - 2 = 0$ (6 marks)

QUESTION FIVE (20 MARKS)

a) Sketch and derive the equation $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ (8 marks)

b) Convert the following parametric equations to Cartesian form and sketch its graph $x = 4 \sec \theta, y = 3 \tan \theta$ where $0 \leq \theta \leq 2\pi$ (4 marks)

c) Find the equation of circle which passes through the point $(20,3), (19,8)$ and $(2, -9)$. Determine its radius and center.   (8 marks)