KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR

FIRST SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF MASTERS (PHYSICS)

COURSE CODE: SPH 810

COURSE TITLE: CLASSICAL MECHANICS

DURATION: 2 HOURS

DATE: 15/12/2022 **TIME: 8-10AM**

INSTRUCTIONS TO CANDIDATES

- Answer **any three** (3) Questions.
- Indicate **answered questions** on the front cover.

Start every question on a new page and make sure question's number is written on each page

This paper consists of **2** printed pages. Please Turn Over

KIBU observes ZERO tolerance to examination cheating

QUESTION ONE [20 Marks]

a) Define Hamilton's principle of least action and explain it [2Marks]

b) Prove that; $-\dot{p_i} = \frac{\partial H}{q_i}; \dot{q_i} = \frac{\partial H}{p_i};$ [6Marks]

c) The Lagrangian for a mass m moving in an inverse- square central force field with characteristic coefficient μ is given by; $L(r, \dot{r}, \theta, \dot{\theta}) = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{\mu m}{r}$. Determine;

i) The generalized momenta [2Marks]

ii) The Hamiltonian [6Marks]

iii) The Hamilton's equations of motion [4Marks]

QUESTION TWO [20 Marks]

a) What is the main problem of calculus of variation? [2 Marks]

b) Show that the shortest distance between two points in a plane is a straight line the two joints [9 Marks]

c) Derive an expression for the brachistochrome problem [9 Marks]

QUESTION THREE [20 Marks]

a) The Hamiltonian of a physical system is given by;

$H = \omega^2 p(q + t)^2$ .Where ω is a constant.

Determine q as a function of time [12 Marks]

b) Prove that the following transformation is canonical;

$P = \frac{1}{2}(p^2 + q^2); Q = tan^{-1}(\frac{q}{p})$ [8 Marks]

QUESTION FOUR [20 Marks]

a) A bead with mass m slides under gravity on a frictionless wire in the shape of a 3-D spiral such that its position in cylindrical coordinates (r, θ,ϕ) is given by;

$r = a\phi; z = b\phi^2)$; where a and b are constants. The kinetic energy of the bead is $T = \frac{m}{2}(\dot{r}^2 + r^2\dot{\phi}^2 + \dot{z}^2)$ and its potential energy is $U = mgz$.

i) Considering the polar angle ϕ as the generalized coordinates; obtain the Lagrangian of the problem [4 Marks]

ii) Obtain the Lagrange equation for the bead [6 Marks]

b) Set up the Lagrangian of a simple pendulum and obtain an equation describing its motion [10Marks]

QUESTION FVE [20 Marks]

a) Using the action-angle formalism, proof that the frequency, ν of a simple one-dimensional harmonic oscillator is given by; [4Marks]

$$\nu = \frac{\sqrt{k/m}}{2\pi}$$

b) Write the Hamiltonian for the 1-dimensional harmonic oscillator of mass, m [4Marks]

c) Write the corresponding Hamilton-Jacobi equation in (b) above [2Marks]

d) Use the Hamilton-Jacobi equation method to obtain the motion of the oscillator [10Marks]

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