(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREES OF BACHELOR OF SCIENCE

COURSE CODE: STA 311

COURSE TITLE: THEORY OF ESTIMATION

DATE: 21/12/22 TIME: 2 PM – 4PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 7 printed pages. Please Turn Over.

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS.

QUESTION ONE (30 MARKS)

1. (a) Differentiate between an estimate and estimator. (2 mks)

   (b) Let X be a normal random variate with unknown mean μ and standard deviation $10km/h$. A random sample of size 10 was selected and yielded $\sum X_i = 890$. Calculate 90% confidence interval for μ (3 mks)

   (c) Let $x_1, x_2, ..., x_n$ be a random sample from a population given by

$$f(x) = \begin{cases} 1, & \beta - \frac{1}{2} < x < \beta + \frac{1}{2} \\ 0, & elsewhere \end{cases}$$

   prove that $\bar{X}$ is a consistent estimator for β. (5 mks)

   (d) If $x_1, ..., x_n$ are the values of a random sample from an exponential population, find the maximum likelihood estimator of its parameter θ (5 mks)

   (e) Let $x_1, \ldots, x_n$ be a random sample from a population with pdf

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1}, x > 0 \\ 0, elsewhere \end{cases}$$

   Show that $t_i = \Pi_{i=1}^{n} X_i$ is a sufficient statistic for θ. (5 mks)

   (f) In a simple random sample of 600 men from city A, 450 are found to smokers. In another random sample of 900 men from city B, 450 are found to be smokers. Construct a 99% confidence interval for the difference in the population proportions. (5 mks)

   (g) Given that $\hat{\theta}$ is unbiased estimator of θ, prove that θ^2 is unbiased estimator of θ^2 iff $Var(\hat{\theta}) = 0$ (5 mks)

   (h) Let a, b and c be a random sample from a normal distribution where both μ and σ are unknown. Which of the following is more efficient estimator for μ

$$\hat{\mu}_1 = \frac{1}{4}a + \frac{1}{2}b + \frac{1}{4}c$$

   or

$$\hat{\mu}_2 = \frac{1}{3}a + \frac{1}{3}b + \frac{1}{3}c$$

   (4 mks)

QUESTION TWO(20 MARKS)

2. (a) Let $x_1, ..., x_n$ be a random sample from the gamma distribution with pdf:

$$f(x; m, \lambda) = \begin{cases} \frac{\lambda(\lambda x)^{m-1}}{\Gamma(m)} e^{-\lambda x}, x > 0, \\ 0, \qquad otherwise \end{cases}$$

Assume m is known, obtain the ML estimator for λ (10 mks)

(b) Show that if $\lim_{n\to\infty} E(\Theta_n) = \theta$ and $\lim_{n\to\infty} var(\Theta_n) = 0$ then the estimator Θ_n is consistent. (5 mks)

(c) Let $(X_1, \ldots, X_n)$ be a random sample of a Poisson random variable X with unknown parameter Λ. Show that $\Lambda_1 = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $\Lambda_2 = \frac{1}{2}(X_1 + X_2)$ are both unbiased estimators of Λ, which estimator is more efficient? (5 mks)

QUESTION THREE (20 MARKS)

3. (a) State precisely as possible the properties of maximum likelihood estimators (2 mks)

(b) A random sample of 50 students out of the total 200 showed a mean of 75 and a standard deviation of 10.

i. What are the 95 percent confidence limits for estimates of the mean of 200 students? (5 mks)

ii. With what degree of confidence could we say that the means of all the 200 students is 75 ± 1? (5 mks)

(c) Let a random sample $X_1, ..., X_n$ be selected from a uniform density over the interval $[0, r]$ where is unknown. Use the method of moments to estimate the parameter r (8 mks)

QUESTION FOUR (20 MARKS)

4. (a) i. When is an estimator t_n of θ said to be sufficient? (2 mks)

   ii. For a normal population $N(\mu, \sigma^2)$ if σ^2 is known, prove that $\bar{X}$ is sufficient estimator for μ (5 mks)

   iii. If μ is known, prove that s^2 is not a sufficient estimator for σ^2 and hence find its sufficient estimator. (3 mks)

   (b) X is a binomial random variable with parameters n (known) and p (unknown). Given a random sample of N observations of X (10 mks)

   i. Compute the method of moments estimator for p

   ii. What will be the method of moments estimator for n and p when both are unknown

QUESTION FIVE(20 MARKS)

5. (a) A random sample is taken from a normal population with mean 0 and variance σ^2. Examine if $s^2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2$ is MVUE for σ^2 (8 mks)

   (b) A random sample is picked from a population whose distribution is given by;

$$f(x; \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$$

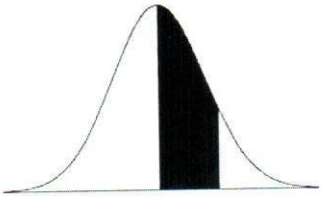
   for $x = 0, 1, 2, \ldots, \infty$

   Taking $\Psi(\lambda) = e^{-\lambda}$, find Cramer-Rao lower bound for t, where t is unbiased estimator for $\Psi(\lambda)$ (7 mks)

   (c) Let $x_1, \ldots, x_n$ be i.i.d. random variable from a Bernoulli distribution with parameter P. Show that $T = \sum_{i}^{n} X_i$ is a sufficient statistic (5 mks)

Appendix: Statistical Tables

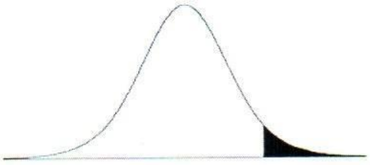
Table 1 Standard normal probabilities (area between 0 and z)



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

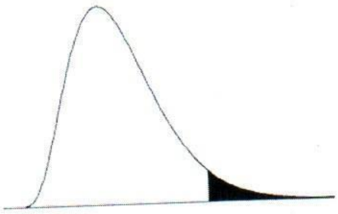
B. Lawal, *Applied Statistical Methods in Agriculture, Health and Life Sciences*, DOI 10.1007/978-3-319-05555-8,

Table 2 Values of t_α in a t distribution with df degrees of freedom. (*shaded area* $P(t > t_\alpha) = \alpha$)



df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	df
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
30	1.310	1.697	2.042	2.457	2.750	30
z	1.282	1.645	1.960	2.326	2.576	z

Table 3 Values of $\chi^2_{\alpha,\mathrm{df}}$ in a chi-square distribution with *df* degrees of freedom (*shaded area* $P(\chi^2 > \chi^2_{\alpha,\mathrm{df}}) = \alpha$)



df	α = .995	α = .990	α = .975	α = .950	α = .050	α = .025	α = .010	α = .005	df
1	0.0000393	0.000157	0.000982	0.00393	3.841	5.024	6.635	7.879	1
2	0.0100	0.0201	0.0506	0.103	5.991	7.378	9.210	10.597	2
3	0.0717	0.115	0.216	0.352	7.815	9.348	11.345	12.838	3
4	0.207	0.297	0.484	0.711	9.488	11.143	13.277	14.860	4
5	0.412	0.554	0.831	1.145	11.070	12.833	15.086	16.750	5
6	0.676	0.872	1.237	1.635	12.592	14.449	16.812	18.548	6
7	0.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278	7
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955	8
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589	9
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188	10
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757	11
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300	12
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819	13
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319	14
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801	15
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267	16
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718	17
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156	18
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582	19
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997	20
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401	21
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796	22
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181	23
24	9.886	10.856	12.401	13.848	36.415	39.364	42.980	45.559	24
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928	25
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290	26
27	11.808	12.879	14.573	16.151	40.113	43.195	46.963	49.645	27
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993	28
29	13.121	14.256	16.047	17.708	42.557	45.722	49.588	52.336	29
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672	30