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(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR
FIRST YEAR FIRST SEMESTER
MAIN EXAMINATION
FOR THE DEGREE OF EDUCATION AND BACHELOR OF
SCIENCE

COURSE CODE: MAA 111 / MAT 121

COURSE TITLE: CALCULUS I / DIFFERENTIAL CALCULUS

DATE: 15/12/2022

TIME: 2.00 P.M - 4.00 P.M

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION 1. [30 MARKS]

[a]. Let $f(x) = \sqrt{\frac{x+2}{x-2}}$ and $g(x) = x^2 - 2$. Find $f \circ g(x)$ and its domain.

[4 MARKS]

[b]. Differentiate the following functions;

i. $f(x) = \frac{\ln x}{x+1}$.

ii. $f(x) = e^x \cos(2x)$.

[6 MARKS]

[c]. The distance S moved by an object in t seconds is given by

$$S = 2t^3 - 13t^2 + 24t + 10.$$

At the point when $t = 4$ seconds, find

i. the velocity of the object.

ii. the acceleration of the object.

[6 MARKS]

[d]. Evaluate the limits,

i. $\lim_{x \rightarrow 9} \frac{x^2 - 81}{\sqrt{x} - 3}$.

ii. $\lim_{x \rightarrow \infty} \frac{3x^2 - x - 10}{2x^2 - 4}$.

[6 MARKS]

[e]. Evaluate the value of $g^{-1}(1)$ given that $g(x) = \sqrt{\frac{x-1}{2}}$

[4 MARKS]

[f]. Using L'Hopital method, find $\lim_{x \rightarrow 0} \frac{\sin 3x}{e^x - 1}$

[4 MARKS]

QUESTION 2. [20 MARKS]

[a .] Find the equation of the normal to the tangent line of the ellipse $4x^2 + y^2 = 8$ at the point $(1, 2)$.

[7 MARKS]

[b .] Find $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3}$

[3 MARKS]

[c .] Show that the $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.

[10 MARKS]

QUESTION 3. [20 MARKS]

[a .] Using ε, δ notation, show that $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = 5$

[5 MARKS]

[b .] Given the curve $y = x^3 - 6x^2 + 9x + 9$

(i). Find the y -intercept.

(ii). Find the turning points of the curve and investigate the nature of each turning point.

(iii). Sketch the curve of $y = x^3 - 6x^2 + 9x + 9$

[11 MARKS]

c. The area of a circular oil slick expands at the rate of $10m^2$ per second. How fast is the radius growing when the area is $100m^2$?

[4 MARKS]

QUESTION 4. [20 MARKS].

[a .] Find the point $(x, 6)$ at which the gradient of the function $x^3 + y^3 = 6xy$ is zero.

[6 MARKS]

[b .] Determine the equation to the tangent line of the parametric functions $x(t) = t^2 + 1$ and $y(t) = (2 + t)^{\frac{1}{2}}$ when $t = 2$.

[10 MARKS]

[c .] Let $f(x) = x^2 + 3x + 2$. Using the limit definition of derivative find $\frac{df(x)}{dx}$.

[4 MARKS]

QUESTION 5. [20 MARKS].

[a .] Sketch the graphs of the functions

(i). $f(x) = |2 - x|$

(ii). $f(x) = \frac{1}{2+x}$

[6 MARKS]

[b .] Calculate $\frac{dy}{dx}$ for

$$y^2 e^x + y \ln x = \cos 2x.$$

[7 MARKS]

[c .] Using Chain rule, find the derivative of the function

$$y = \ln(x + \sqrt{x^2 + 1})$$

[4 MARKS]

[d .] Show that the function $f(x) = \frac{x^3 - 8}{x - 2}$ is continuous in the domain $[2, \infty)$.

[3 MARKS]