KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR

FIRST SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF MASTERS (PHYSICS)

COURSE CODE: SPH 814

COURSE TITLE: STATISTICAL MECHANICS

DURATION: 2 HOURS

DATE: 13/12/2022 **TIME: 2-4PM**

INSTRUCTIONS TO CANDIDATES

- Answer any **Three (3)** Questions.
- Indicate **answered questions** on the front cover.

Start every question on a new page and make sure question's number is written on each page

This paper consists of **2** printed pages. Please Turn Over

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QUESTION ONE [20 Marks]

a) Show that entropy, S is an extensive property. [4mks]

b) In classical micro-canonical ensemble the entropy of an ideal gas of volume V and number of particles N is given as;

$$S(U,VN) = Nkln\left[v\left(\frac{4\pi mE^{3/2}}{3h^2N}\right)\right] + \frac{3}{2}Nk$$

where the terms have their usual meanings.

Use the above expression to determine;

i) Temperature, T [4mks]

ii) Internal energy, U [4mks]

iii) Heat capacity, Cv [4mks]

iv) Equation of state [4mks]

QUESTION TWO [20 Marks]

a) Show that the partition function for a classical ideal gas is given by; [10mks]

$$Q_N(V,T) = \frac{1}{N!}[\frac{V}{h^3}(2\pi mkT)^{\frac{3}{2}}]^N$$

b) Consider an ideal gas of N non interacting indistinguishable particles placed in a volume V. The single particle Hamiltonian is $H = \frac{p^2}{2m}$, with p the absolute value of the momentum and m the mass of each particle. Prove the following relations [10mks]

$$\frac{S(T,V,N)}{Nk} = ln\left(\frac{Q_1(T,V)}{N}\right) + T\left(\frac{\partial Q_1(T,V)}{\partial T}\right)_V + 1$$

$$\frac{S(T,P,N)}{Nk} = ln\left(\frac{Q_1(T,P,N)}{N}\right) + T\left(\frac{\partial Q_1(T,P,N)}{\partial T}\right)_{PN}$$

Where S is the entropy, P the pressure, and Q1 is the canonical partition function of the single particle.

QUESTION THREE [20 Marks]

a) Using the first law of thermodynamics, write the chemical potential in terms of energy derivatives. Repeat this computation writing it in terms of entropy derivatives. Using the Sackur-Tetrode formula for the entropy;

$$S(U,VN) = Nk\left\{\frac{5}{2} - ln\left[\left(\frac{3\pi\hbar^2}{m}\right)\frac{N^{5/2}}{VU^{3/2}}\right]\right\}$$

Show that these two formulae for the chemical potential lead to the same result that is found using the formalism of the canonical ensemble [10mks]

b) Consider a free gas with N-particles and internal energy U inside a container of volume V. Starting with the Sucker-Tetrode formula for entropy given below;

$$S(U,VN) = Nk\left\{\frac{5}{2} - ln\left[\left(\frac{3\pi\hbar^2}{m}\right)\frac{N^{5/2}}{VU^{3/2}}\right]\right\}$$

Find the Helmholtz Free energy F, internal energy U, enthalpy H, and the Gibbs potential Φ, temperature T and pressure P of the gas and hence equation of state. [10mks]

QUESTION FOUR [20 Marks]

a) Define phase space and write down the equations of motion of a phase point considering the motion of an oscillator in phase space. [4mks]

b) Using Hamilton's equations show that the path of the body falling under gravity is a parabola. [6mks]

c) Show that the orbit in phase space of a simple linear harmonic oscillator is an ellipse and that its period, T in phase space is equal to the area of the phase ellipse divided by the energy, E of the oscillator. [10mks]

QUESTION FVE [20 Marks]

a) Differentiate between a Fermi system and a Bose system. [4mks]

b) The grand potential for an ideal gas of quantum particles i.e

$$\Phi(T,V,\mu) = -kTlogZ = U - TS - \mu N = -PV$$

Use this expression to show that the equation of state of an ideal Bose gas if given by;

$\frac{P}{kT} = \frac{1}{\lambda^3} g_{5/2}(Z)$ [16mks]

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