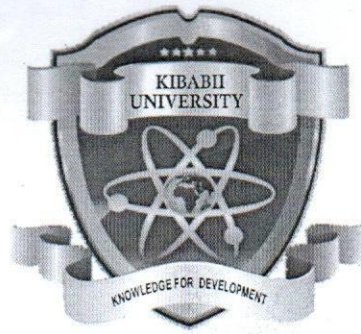


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*(Knowledge for Development)*

**KIBABII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**  
**2022/2023 ACADEMIC YEAR**  
**THIRD YEAR FIRST SEMESTER**  
**MAIN EXAMINATION**

**FOR THE DEGREE OF BACHELOR OF EDUCATION AND**  
**BACHELOR OF SCIENCE**

**COURSE CODE: MAT 321/MAA 311**

**COURSE TITLE: ODE I**

**DATE: 13/12/2022**

**TIME: 2 PM -4 PM**

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**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

### QUESTION ONE (30 MARKS)

a) Classify the following differential equations as to order, degree and linearity

i)  $2(y'')^3 - 3x(y''')^2 + 5x = y$  (3 mks)

ii)  $\frac{d^4y}{dx^4} - 7\frac{d^2y}{dx^2} + 6y = 12$  (3 mks)

b) Show that it is homogeneous and solve the differential equation

$(x + y)^2 dx + 3xy dy = 0$  (6 mks)

c) Obtain the differential equation having a solution as

$y = Ae^{-4x} - Be^{2x} + C$  (6 mks)

d) Solve the non-homogeneous differential equation

$2y'' + 3y' + y = x^2 + 3\sin x$  (6 mks)

e) The temperature of a car engine by the time it is shut off is  $220^\circ C$ . The surrounding air temperature is  $35^\circ C$ . After 12 seconds have elapsed, the temperature of the engine is  $150^\circ C$

(i) How long will it take the engine to cool to  $60^\circ C$  (4 mks)

(ii) Find the engine temperature after 70 seconds (2 mks)

### QUESTION TWO [20 MARKS]

a) Test for exactness and solve the differential equation

$(2y^3 - x^2y - 2x + 3)dy - (xy^2 + 2y)dx = 0$   $y(0) = 1$  (8 mks)

b) Show that  $\frac{dy}{dx} = \frac{2x-x^2}{3+y^3}$  is separable and hence find its particular solution

under the Condition  $y(2) = -1$  (5 mks)

c) A man sitting on an electricity power post 20 M high throws a nail upward with a speed of 30M/S

(i) How long does the nail stay in the air (4 mks)

(ii) With what speed does the nail hit the ground (3 mks)

**QUESTION THREE [20 MARKS]**

- (a) Obtain a differential equation corresponding to the following operator equation

$$(2D + 3x)(2D - 3x)y = 0 \quad (5 \text{ mks})$$

- (b) Solve the equation  $\frac{dy}{dx} - 3xy = 3xe^{-x^2}$  (7 mks)

- (c) Given the differential equation  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$

Verify that the functions  $e^x \cos 2x$  and  $e^x \sin 2x$  are linearly independent solutions of the homogeneous part of the differential equation. (8 mks)

**QUESTION FOUR [20 MARKS]**

- a) Solve the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}, \quad (7 \text{ mks})$$

- b) Solve the linear fractional differential equation

$$(x + 2y + 1)dx - (2x + 4y + 3)dy = 0 \quad (6 \text{ mks})$$

- c) Solve the equation by method of variation of parameter

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 2e^x \quad (7 \text{ mks})$$

**QUESTION FIVE [20 MARKS]**

- (a) The sum of Kshs 2800 is invested at a rate of 11% per annum compounded continuously.

What will be the amount after 8 years? (4 mks)

- (b) Using the integrating factor solve the differential equation

$$x(y + x + 1)dy + y(y + 3x + 2)dx = 0 \quad (9 \text{ mks})$$

- (c) Solve the following Bernoulli's equation

$$2xy \frac{dy}{dx} = y^2 - 2x^3, y(1) = -2 \quad (7 \text{ mks})$$