230

(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 321/MAA 311

COURSE TITLE: ODE I

DATE: 13/12/2022 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Classify the following differential equations as to order, degree and linearity

i) $2(y^{II})^3 - 3x(y^{III})^2 + 5x = y$ (3 mks)

ii) $\frac{d^4y}{dx^4} - 7\frac{d^2y}{dx^2} + 6y = 12$ (3 mks)

b) Show that it is homogeneous and solve the differential equation

$(x+y)^2dx + 3xydy = 0$ (6 mks)

c) Obtain the differential equation having a solution as

$y = Ae^{-4x} - Be^{2x} + C$ (6 mks)

d) Solve the non-homogeneous differential equation

$2y^{II} + 3y^{I} + y = x^2 + 3sinx$ (6 mks)

e) The temperature of a car engine by the time it is shut off is 220^0C. The surrounding air temperature is 35^0C. After 12 seconds have elapsed, the temperature of the engine is 150^0C

(i) How long will it take the engine to cool to 60^0C (4 mks)

(ii) Find the engine temperature after 70 seconds (2 mks)

QUESTION TWO [20 MARKS]

a) Test for exactness and solve the differential equation

$(2y^3 - x^2y - 2x + 3)dy - (xy^2 + 2y)dx = 0 \quad y(0) = 1$ (8 mks)

b) Show that $\frac{dy}{dx} = \frac{2x-x^2}{3+y^3}$ is separable and hence find its particular solution under the Condition $y(2) = -1$ (5 mks)

c) A man sitting on an electricity power post 20 M high throws a nail upward with a speed of 30M/S

(i) How long does the nail stay in the air (4 mks)

(ii) With what speed does the nail hit the ground (3 mks)

QUESTION THREE [20 MARKS]

(a) Obtain a differential equation corresponding to the following operator equation

$(2D + 3x)(2D - 3x)y = 0$ (5 mks)

(b) Solve the equation $\frac{dy}{dx} - 3xy = 3xe^{-x^2}$ (7 mks)

(c) Given the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$

Verify that the functions $e^x cos2x$ and $e^x sin2x$ are linearly independent solutions of the homogeneous part of the differential equation. (8 mks)

QUESTION FOUR [20 MARKS]

a) Solve the differential equation

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}$, (7 mks)

b) Solve the linear fractional differential equation

$(x + 2y + 1)dx - (2x + 4y + 3)dy = 0$ (6 mks)

c) Solve the equation by method of variation of parameter

$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 2e^x$ (7 mks)

QUESTION FIVE [20 MARKS]

(a) The sum of Kshs 2800 is invested at a rate of 11% per annum compounded continuously. What will be the amount after 8 years? (4 mks)

(b) Using the integrating factor solve the differential equation

$x(y + x + 1)dy + y(y + 3x + 2)dx = 0$ (9 mks)

(c) Solve the following Bernoulli's equation

$2xy\frac{dy}{dx} = y^2 - 2x^3$, $y(1) = -2$ (7 mks)