

13-5



*(Knowledge for development)*

**KIBABII UNIVERSITY  
(KIBU)**

**UNIVERSITY EXAMINATIONS  
2022/2023 ACADEMIC YEAR**

**END OF SEMESTER EXAMINATIONS  
FIRST YEAR FIRST SEMESTER**

**FOR THE DEGREE IN  
(INFORMATION TECHNOLOGY/ COMPUTER  
SCIENCE)**

**COURSE CODE: BIT 111/CSC 112**

**COURSE TITLE: DISCRETE STRUCTURES**

**DATE: 13/12/2022**

**TIME: 9.00 A.M- 11.00 A.M.**

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**INSTRUCTIONS**

**ANSWER QUESTIONS ONE AND ANY OTHER TWO.**

**QUESTION ONE (COMPULSORY)****[30 MARKS]**

- a. Given non-empty sets A, B, C, and that they are not disjoint. Define the inclusive-exclusive principle on sets A, B and C. **[2 marks]**
- b. Given two sets  $A = \{a_1, a_2, a_3\}$  and  $B = \{b_1, b_2, b_3\}$ , define the Cartesian product of A and B and show that Cartesian product is not commutative. **[3 marks]**
- c. Determine truth value of the statement: If  $2 + 5 = 25$  then Uganda is in Indian Ocean. **[2 marks]**
- d. Find:
- The value of n if  ${}_n P_3 = 5 \cdot {}_n P_2$  **[2 marks]**
  - $C(11,7)$  **[2 marks]**
- e. Prove by the method of induction, that for all  $n \in N$ ,
- $$1.2.3 + 2.3.4 + 3.4.5 + \dots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$
- [4 marks]**
- f. Let  $f: \mathbf{R} \rightarrow \mathbf{R}$  be defined by  $f(x) = 2x^3 + 51$ . Find the inverse of  $f(x)$ . **[2 marks]**
- g. Let  $A = \{1, 2, 3, 4, 5\}$ . Determine the truth value of the following statement:  
 $(\exists x \in A)(x + 3 = 10)$  **[2 marks]**
- h. In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories? **[3 marks]**
- i. Using Euclidean algorithm find the GCD of 1215 and 4551 hence or otherwise find the value of x and y in  $x(1215) + y(4551) = \text{gcd}(1215, 4551)$ . **[4 marks]**
- a. Let R and S be the following relations on  $A = \{1, 2, 3\}$ :  $R = \{(1, 1), (1, 2), (2, 3), (3, 1), (3, 3)\}$ ,  $S = \{(1, 2), (1, 3), (2, 1), (3, 3)\}$  Find:
- $(R \cup S)$  and  $(R \cap S)$  **[2 marks]**
  - $R \circ S$  **[2 marks]**

**QUESTION TWO****[20 MARKS]**

a. Differentiate between a function and a relation as used in the study of discrete structures.

**[2 marks]**

i. Let  $R$  be the relation on  $\mathbb{N}$  defined by  $x + 3y = 12$ , i.e.  $R = \{(x, y) \mid x + 3y = 12\}$ . Write  $R$  as a set of ordered pairs and determine the composition relation  $R \circ R$ .

**[4 marks]**

b. Given  $A = \{1, 2, 3, 4\}$  and  $B = \{x, y, z\}$ . Let  $R$  be the following relation from  $A$  to  $B$ :

$$R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}$$

i. Find the inverse relation  $R^{-1}$  of  $R$ .

**[2 marks]**

ii. Determine the matrix of the relation.

**[2 marks]**

iii. Draw the arrow diagram or digraph of  $R$ .

**[2 marks]**

c. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = \frac{2x+1}{x}$  and  $g(x) = \frac{x^2-2}{x-2}$ .

Find

i. Domain and range of  $f(x)$

**[2 marks]**

ii.  $g \circ f$

**[2 marks]**

iii.  $f(g(3))$

**[2 marks]**

iv. If  $g(x)$  is one-to-one mapping or onto.

**[2 marks]****QUESTION THREE****[20 MARKS]**

a. Given  $U$  as a set of English alphabets and sets  $A$ ,  $B$  and  $C$  formed from distinct characters of the word "generosity", "crocodile" and "programming" respectively.

Find:

i.  $n(A \cup B \cup C)$

**[2 marks]**

ii.  $(A \cap B \cap C)^c$

**[2 marks]**

b. The students who stay in hostel were asked whether they had a textbook and digest in their rooms. The results showed that 750 students had a textbook, 250 did not have a textbook, 225 had a digest and 100 had neither a textbook nor a digest. Find:

i. The number of students in hostel

**[2 marks]**

ii. How many have both a textbook and digest

**[2 marks]**

iii. How many have only a digest.

**[2 marks]**

c. Determine the truth value of each of the following statements where  $U = \{1, 2, 3\}$  is the universal set:

- i.  $\exists x \forall y, x^2 < y + 1$  [2 marks]
- ii.  $\forall x \forall y, x^2 + y^2 < 12$  [2 marks]
- d. i. You are given the propositions, **p**: the students are rowdy, **q**: the situation is violent and **r**: lessons are cancelled. Represent the following statement “**it’s not the case that when students are rowdy and the situation is violent then lessons are cancelled**” symbolically and draw its truth table. [4 marks]
- ii. Give the symbolic form of “**some men are greedy**” [2 marks]

#### QUESTION FOUR

[20 MARKS]

- a. Differentiate between permutation and combination. [2 marks]
- b. A history class contains 8 male students and 6 female students. Find the number  $n$  of ways that the class can elect:
- i. 2 class representatives, 1 male and 1 female. [2 marks]
- ii. 1 president and 1 vice president. [2 marks]
- c. Consider all integers from 1 up to and including 100. Find the number of them that are even or the cube of an integer [2 marks]
- d. In a class of 30 students, 10 got A on the first test, 9 got A on a second test, and 15 did not get an A on either test. Find: the number of students who got:
- i. an A on both tests [2 marks]
- ii. an A on the first test but not the second [2 marks]
- iii. an A on the second test but not the first [2 marks]
- e. Using the principle of mathematical induction show that:

$$\sum_{r=0}^n 3^r = \frac{3^{2n+n}-1}{2} \text{ for } \forall n \in \mathbb{N}.$$

[6 marks]

#### QUESTION FIVE

[20 MARKS]

- a. What is a logic gate? [1 mark]
- b. Using A and B as inputs, draw logic gates and a truth table for:
- OR gate [3 marks]
- AND gate [3 marks]
- NOR gate [3 marks]
- XOR gate [3 marks]

c. Write down the output denoted by Q in Figure1 below.

[2 marks]

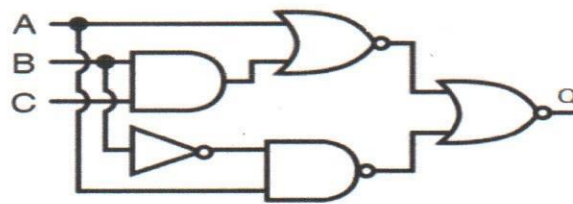


Figure 1: Logic Circuit

d. Prove that  ${}_n P_r = (n-r+1) \cdot {}_n P_{(r-1)}$

[5 marks]