(Knowledge for development)

KIBABII UNIVERSITY

(KIBU)

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

END OF SEMESTER EXAMINATIONS

FIRST YEAR FIRST SEMESTER

FOR THE DEGREE IN

(INFORMATION TECHNOLOGY/ COMPUTER SCIENCE)

COURSE CODE: BIT 111/CSC 112

COURSE TITLE: DISCRETE STRUCTURES

DATE: 13/12/2022 TIME: 9.00 A.M- 11.00 A.M.

INSTRUCTIONS

ANSWER QUESTIONS ONE AND ANY OTHER TWO.

QUESTION ONE (COMPULSORY) **[30 MARKS]**

a. Given non-empty sets A, B, C, and that they are not disjoint. Define the inclusive-exclusive principle on sets A, B and C. **[2 marks]**

b. Given two sets A= $\{a_1, a_2, a_3\}$ and B= $\{b_1, b_2, b_3\}$, define the Cartesian product of A and B and show that Cartesian product is not commutative. **[3 marks]**

c. Determine truth value of the statement: If 2 + 5 = 25 then Uganda is in Indian Ocean. **[2 marks]**

d. Find:

   i. The value of n if ${}_nP_3 = 5*{}_nP_2$ **[2 marks]**

   ii. C(11,7) **[2 marks]**

e. Prove by the method of induction, that for all $n \in N$,

   $1.2.3 + 2.3.4 + 3.4.5 + \ldots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$ **[4 marks]**

f. Let f: **R** → **R** be defined by $f(x) = 2x^3 + 51$.Find the inverse of $f(x)$. **[2 marks]**

g. Let A = {1, 2, 3, 4, 5}. Determine the truth value of the following statement:
   $(\exists x \in A)(x + 3 = 10)$ **[2 marks]**

h. In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories? **[3 marks]**

i. Using Euclidean algorithm find the GCD of 1215 and 4551 hence or otherwise find the value of **x** and **y** in **x**(1215)+ **y**(4551)=gcd(1215, 4551). **[4 marks]**

a. Let R and S be the following relations on A = {1, 2, 3}: R = {(1, 1), (1, 2), (2, 3), (3, 1), (3, 3)}, S = {(1, 2), (1, 3), (2, 1), (3, 3)} Find:

   i. $(R \cup S)$ and $(R \cap S)$ **[2 marks]**

   ii. $R \circ S$ **[2 marks]**

QUESTION TWO [20 MARKS]

a. Differentiate between a function and a relation as used in the study of discrete structures. **[2 marks]**

i. Let R be the relation on N defined by $x + 3y = 12$, i.e. $R = \{(x, y) \mid x + 3y = 12\}$. Write R as a set of ordered pairs and determine the composition relation $R \circ R$. **[4 marks]**

b. Given $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$. Let R be the following relation from A to B:

$R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}$

i. Find the inverse relation R^{-1} of R. **[2 marks]**

ii. Determine the matrix of the relation. **[2 marks]**

iii. Draw the arrow diagram or digraph of R. **[2 marks]**

c. Let f: $\mathbf{R} \rightarrow \mathbf{R}$ and g: $\mathbf{R} \rightarrow \mathbf{R}$ be defined by $f(x) = \frac{2x+1}{x}$ and $g(x) = \frac{x2-2}{x-2}$.

Find

i. Domain and range of **f**(x) **[2 marks]**

ii. $g \circ f$ **[2 marks]**

iii. f(g(3)) **[2 marks]**

iv. If g(x) is one-to-one mapping or onto. **[2 marks]**

QUESTION THREE [20 MARKS]

a. Given U as a set of English alphabets and sets A, B and C formed from distinct characters of the word "**generosity**", "**crocodile**" and "**programming**" respectively.

Find:

i. n(AUBU) **[2 marks]**

ii. $(A \cap B \cap C)^c$ **[2 marks]**

b. The students who stay in hostel were asked whether they had a textbook and digest in their rooms. The results showed that 750 students had a textbook, 250 did not have a textbook, 225 had a digest and 100 had neither a textbook nor a digest. Find:

i. The number of students in hostel **[2 marks]**

ii. How many have both a textbook and digest **[2 marks]**

iii. How many have only a digest. **[2 marks]**

c. Determine the truth value of each of the following statements where $U = \{1, 2, 3\}$ is the universal set:

i. $\exists x \forall y, x^2 < y + 1$ **[2 marks]**

ii. $\forall x \forall y, x^2 + y^2 < 12$ **[2 marks]**

d. i. You are given the propositions, **p**: the students are rowdy, **q**: the situation is violent and **r**: lessons are cancelled. Represent the following statement "**it's not the case that when students are rowdy and the situation is violent then lessons are cancelled**" symbolically and draw its truth table. **[4 marks]**

ii. Give the symbolic form of "**some men are greedy**" **[2 marks]**

QUESTION FOUR **[20 MARKS]**

a. Differentiate between permutation and combination. **[2 marks]**

b. A history class contains 8 male students and 6 female students. Find the number *n* of ways that the class can elect:

i. 2 class representatives, 1 male and 1 female. **[2 marks]**

ii. 1 president and 1 vice president. **[2 marks]**

c. Consider all integers from 1 up to and including 100. Find the number of them that are even or the cube of an integer **[2 marks]**

d. In a class of 30 students, 10 got A on the first test, 9 got A on a second test, and 15 did not get an A on either test. Find: the number of students who got:

i. an A on both tests **[2 marks]**

ii. an A on the first test but not the second **[2 marks]**

iii. an A on the second test but not the first **[2 marks]**

e. Using the principle of mathematical induction show that:

$\sum_{r=0}^{n} 3^r = \frac{3^{2n+n}-1}{2}$ for $\forall n \epsilon N$. **[6 marks]**

QUESTION FIVE **[20 MARKS]**

a. What is a logic gate? **[1 mark]**

b. Using A and B as inputs, draw logic gates and a truth table for:

OR gate **[3 marks]**

AND gate **[3 marks]**

NOR gate **[3 marks]**

XOR gate **[3 marks]**

c. Write down the output denoted by Q in Figure1 below. **[2 marks]**

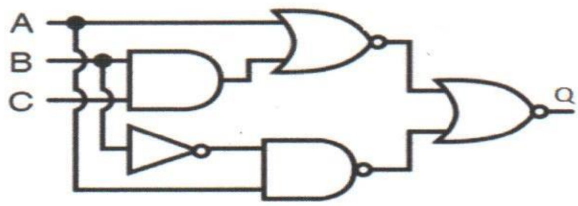

Figure 1: Logic Circuit

d. Prove that $_nP_r = (n-r+1)^* \, nP_{(r-1)}$ **[5 marks]**