



KIBABII UNIVERSITY

**UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR**

**FIRST SEMESTER
MAIN EXAMINATIONS**

FOR THE DEGREE OF MASTERS (PHYSICS)

COURSE CODE: SPH 814

COURSE TITLE: STATISTICAL MECHANICS

DURATION: 2 HOURS

DATE: 13/12/2022

TIME: 2-4PM

INSTRUCTIONS TO CANDIDATES

- Answer any **Three (3)** Questions.
 - Indicate **answered questions** on the front cover.
- Start every question on a new page and make sure question's number is written on each page

This paper consists of **2** printed pages. Please Turn Over

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QUESTION ONE [20 Marks]

- a) Show that entropy, S is an extensive property. [4mks]
 b) In classical micro-canonical ensemble the entropy of an ideal gas of volume V and number of particles N is given as;

$$S(U, VN) = Nk \ln \left[v \left(\frac{4\pi m E^{3/2}}{3h^2 N} \right) \right] + \frac{3}{2} Nk$$

where the terms have their usual meanings.

Use the above expression to determine;

- i) Temperature, T [4mks]
 ii) Internal energy, U [4mks]
 iii) Heat capacity, Cv [4mks]
 iv) Equation of state [4mks]

QUESTION TWO [20 Marks]

- a) Show that the partition function for a classical ideal gas is given by;

$$Q_N(V, T) = \frac{1}{N!} \left[\frac{V}{h^3} (2\pi mkT)^{3/2} \right]^N \quad [10mks]$$

- b) Consider an ideal gas of N non interacting indistinguishable particles placed in a volume V. The single particle Hamiltonian is $H = \frac{p^2}{2m}$, with p the absolute value of the momentum and m the mass of each particle. Prove the following relations [10mks]

$$\frac{S(T, V, N)}{Nk} = \ln \left(\frac{Q_1(T, V)}{N} \right) + T \left(\frac{\partial Q_1(T, V)}{\partial T} \right)_V + 1$$

$$\frac{S(T, P, N)}{Nk} = \ln \left(\frac{Q_1(T, P, N)}{N} \right) + T \left(\frac{\partial Q_1(T, P, N)}{\partial T} \right)_{PN}$$

Where S is the entropy, P the pressure, and Q1 is the canonical partition function of the single particle.

QUESTION THREE [20 Marks]

- a) Using the first law of thermodynamics, write the chemical potential in terms of energy derivatives. Repeat this computation writing it in terms of entropy derivatives. Using the Sackur-Tetrode formula for the entropy;

$$S(U, VN) = Nk \left\{ \frac{5}{2} - \ln \left[\left(\frac{3\pi \hbar^2}{m} \right) \frac{N^{5/2}}{VU^{3/2}} \right] \right\}$$

Show that these two formulae for the chemical potential lead to the same result that is found using the formalism of the canonical ensemble [10mks]

- b) Consider a free gas with N-particles and internal energy U inside a container of volume V. Starting with the Sucker-Tetrode formula for entropy given below;

$$S(U, VN) = Nk \left\{ \frac{5}{2} - \ln \left[\left(\frac{3\pi\hbar^2}{m} \right) \frac{N^{5/2}}{VU^{3/2}} \right] \right\}$$

Find the Helmholtz Free energy F, internal energy U, enthalpy H, and the Gibbs potential Φ , temperature T and pressure P of the gas and hence equation of state.

[10mks]

QUESTION FOUR [20 Marks]

- a) Define phase space and write down the equations of motion of a phase point considering the motion of an oscillator in phase space. [4mks]
- b) Using Hamilton's equations show that the path of the body falling under gravity is a parabola. [6mks]
- c) Show that the orbit in phase space of a simple linear harmonic oscillator is an ellipse and that its period, T in phase space is equal to the area of the phase ellipse divided by the energy, E of the oscillator. [10mks]

QUESTION FVE [20 Marks]

- a) Differentiate between a Fermi system and a Bose system. [4mks]
- b) The grand potential for an ideal gas of quantum particles i.e

$$\Phi(T, V, \mu) = -kT \log Z = U - TS - \mu N = -PV$$

Use this expression to show that the equation of state of an ideal Bose gas if given by;

$$\frac{P}{kT} = \frac{1}{\lambda^3} g_{5/2}(Z) \quad [16mks]$$

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