(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAP 321**

COURSE TITLE: **REAL ANALYSIS III**

DATE: 06/09/2022 **TIME:** 9:00 AM - 11:00 AM

Answer question ONE and any other TWO Questions

TIME: 2 Hours

QUESTION ONE (20 MARKS)

a) Define the following terms

   i. Sequence

   ii. Riemann-Integrable function

   iii. Fourier series

b) Show that if p^* is a refinement of p, then $L(p, f, \alpha) \leq L(p^*, f, \alpha')$ and $U(p^*, f, \alpha') \leq U(p, f, \alpha)$

c) Show that $f \in R(\alpha)$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition p such that $U(p, f, \alpha) - L(p, f, \alpha) < \varepsilon$

QUESTION TWO (20 MARKS)

a) State the five properties of the integral

b) Suppose $c_n \geq 0$ for 1,2,3,.... $\sum c_n$ converges, $\{s_n\}$ is a sequence of distinct points on (a,b) and $\alpha(x) = \sum_{n=1}^{\infty} c_n I(x - s_n)$. Let f be continuous on [a,b] then show $\int_a^b f d\alpha = \sum_{n=1}^{\infty} c_n f(s_n)$

c) Show that if $A = \sum_{n=1}^{\infty} a_n$ and $B = \sum_{n=1}^{\infty} b_n$ both exist and are finite, then

   i. $\sum_{n=1}^{\infty} (a_n + b_n) = A + B$ is monotonic, that is $\mu(E) \leq \mu(F)$ whenever E and F are sets in R such that $E \subset F$

   ii. $\sum_{n=1}^{\infty} k a_n = k \sum_{n=1}^{\infty} a_n = kA \ (k \in \mathbb{R})$

QUESTION THREE (20 MARKS)

a) Show that if $\sum_{n=1}^{\infty} \frac{1}{n!}$ Converges

b) State the comparison test for series of nonnegative terms

c) Show that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

QUESTION FOUR (20 MARKS)

a) Show that if $\lim_{n \to \infty} a_n \neq 0$ or if $\lim_{n \to \infty} a_n$ fails to exist, then $\sum_{n=1}^{\infty} a_n$ diverges

b) Given the function y=f(x) obtained by introducing the continuous variable x in place of the discrete variable in the nth trm of the positive series $\sum_{n=1}^{\infty} a_n$ be a decreasing function of x for $x \geq 1$, show that the series and the integral $\int_1^{\infty} f(x)dx$ both converge or both diverge

c) Show that if p is a real constant, the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p < 1$

QUESTION FIVE (20 MARKS)

a) Assume α increases monotonically and $\alpha' \in R$ on $[a, b]$, let f be a bounded real function on $[a, b]$. show that $f \in R(\alpha)$ if and only if $f\alpha' \in R$. In that case, $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$

b) Suppose φ is strictly increasing continuous function that maps an interval $[a, b]$, suppose α is monotonically increasing on $[a, b]$ and $f \in R(\alpha)$ on $[a, b]$. Define β and g on $[A, B]$ by $\beta(y) = \alpha(\varphi(y))$ *and* $g(y) = f(\varphi(y))$ show that $\int_A^B d\beta = \int_a^b d\alpha$