



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR
3RD YEAR FIRST SEMESTER
MAIN EXAMINATION

**FOR THE DEGREE OF BACHELOR OF EDUCATION AND
BACHELOR OF SCIENCE**

COURSE CODE: MAP 314

COURSE TITLE: NUMBER THEORY

DATE: 14/12/2022

TIME: 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **2 Printed Pages**. Please Turn Over.

QUESTION 1: 30 MARKS

- a) Show that an integer p is prime iff it is irreducible. (3 marks)
- b) Prove that $2^0 + 2^1 + 2^2 + \dots + 2^{n-1} = 2^n - 1$. (7 marks)
- c) Find the general solution to the following Diophantine equation $8x + 14y + 5z = 11$ (4 marks)
- d) Find all the prime numbers up to 100 using sieve of Eratosthenes. (4 marks)
- e) Prove that $\sqrt[3]{2}$ is not a rational number. (7 marks)
- f) Define a Diophantine problem (2 marks)
- g) Prove that $1+3+5+\dots+(2n-1)=n^2$ (3 marks)

QUESTION 2: 20 MARKS

- a) Find $17^{341} \pmod{5}$. (6 marks)
- b) Prove that if x is even, then $x^2 + 2x + 4$ is divisible by 4. (6 marks)
- c) Prove that if $n \in \mathbb{Z}$, then n^2 does not have a remainder of 2 or 3 when it is divided by (8 marks)

QUESTION 3: 20 MARKS

- a) Sharon buys large shirts for \$ 18 each and small shirts for \$ 11 each. The shirts cost a total of \$ 1188. What is the smallest total number of shirts she could have bought? (11 marks)
- b) The greatest common divisor of any two numbers a and b , which are not simultaneously zero, exists and is unique. It is the biggest among the common divisors of a and b . Prove. (5 marks)
- c) Show that if $a \equiv b \pmod{n}$ then $b = a + nq$ for some integer q , and conversely. (4 marks)

QUESTION 4: 20 MARKS

- a) Let p be prime, and suppose p does not divide a . then $a^{p-1} = 1 \pmod{p}$. (10 marks)
- b) Prove that $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}$ for every positive integer n . (10 marks)

QUESTION 5: 20 MARKS

- a) If $n \in \mathbb{Z}$, prove that $0 \cdot n = 0$ (5 marks)
- b) Solve for x : $5x \equiv 1 \pmod{12}$ (5 marks)
- c) Let p be a prime number and suppose that p divides ab where a and b are integers. Then either p divides a or p divides b (or both). Prove. (10 marks)