(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (RENEWABLE ENERGY)

COURSE CODE: **MAT 251**

COURSE TITLE: ENGINEERING MATHEMATICS I

DATE: 13/12/2022 **TIME: 9:00 AM – 11:00 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

(a) If $z_1 = 3-4i$ and $z_2 = -2+3i$, find the value of $2z_1 - 3z_2$ **(3mks)**

(b) Find the distance between the complex numbers $2+i$ and $1-2i$ **(4mks)**

(c) Prove that

$$\frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)} = \tan A$$ **(4mks)**

(d) Find the general solution of the following homogenous differential equation

$y' + 5y = 0$. **(5mks)**

(e) Write $\sum_{n=1}^{\infty} ar^{n-1}$ as a series that starts at $n = 0$. **(2mks)**

(f) If $A = \begin{bmatrix} 2 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 4 \\ 2 & 6 & 7 \end{bmatrix}$, find $3A-2B$. **(4mks)**

(g) Find the Tailor series for $f(x) = e^x$ about $x = 0$. **(4mks)**

(h) Given the series $\sum_{n=0}^{\infty} \frac{1}{2^n}$, determine the following;

(i) The partial sum s_n. **(2mks)**

(ii) The sum of the series **(2mks)**

QUESTION TWO (20 MARKS)

Find the eigenvalues and eigenvectors for the following matrix; **(20mks)**

$$A = \begin{bmatrix} 2 & 7 \\ -1 & -6 \end{bmatrix}$$

QUESTION THREE (20 MARKS)

(a) Determine the general solution of the differential equation;

$$\frac{dy}{dt} + 3t^2 y = 6t^2$$ **(8mks)**

(b) Solve the following initial value problem concerning homogenous differential equation.

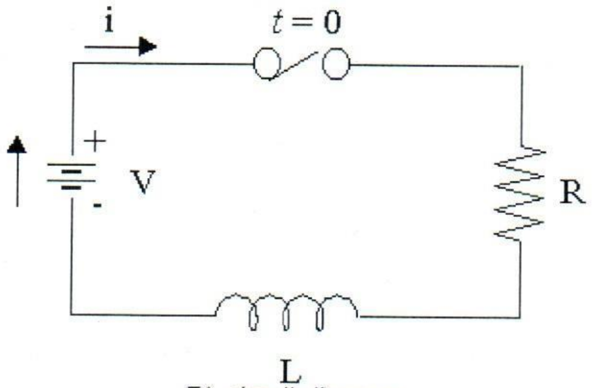
(i) $y' + y = 0, y(0) = 4$ **(6mks)**

(ii) $y' + y \sin t = 0, y(\pi) = 1$ **(6mks)**

QUESTION FOUR (20 MARKS)

(a) A radioactive substance obeys the equation $y' = ky$ where $k < 0$ and y is the mass of the substance at time t. Suppose that initially, the mass of the substance is $y(0) = M > 0$. At what time does half the mass remain? **(10mks)**

(b)



RL circuit diagram

The RL circuit shown above has a resistor and an inductor connected in series. A constant voltage V is applied when the switch is closed. The (variable) voltage across the resistor is given by $V_R = iR$. The (variable) voltage across the inductor is given by $Ri + L\frac{di}{dt} = V$. Once the switch is closed, the current in the circuit is not constant. Instead, it will build up from zero to some steady state. Prove that the current in the circuit is given by $i = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$ **(10mks)**

QUESTION FIVE (20 MARKS)

(a) Compute the following series, if it converges.

(i) $\sum_{n=0}^{\infty}\left(\frac{4}{(-3)^n} - \frac{3}{3^n}\right)$ **(5mks)**

(ii) $\sum_{n=0}^{\infty}\frac{3^n}{5^{n+1}}$ **(5mks)**

(b) Find the Maclaurin series for $f(x) = \frac{1}{x}$ **(6mks)**

(c) Given $f(x) = e^x$, find the following the Taylor series of f centred at 3. **(4mks)**