KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF B.SC (PHYSICS)

COURSE CODE: **SPH312**

COURSE TITLE: **CLASSICAL MECHANICS II**

DURATION: **2 Hours**

DATE: 20/12/2022 **TIME**: 9:00-11:00AM

INSTRUCTIONS TO CANDIDATES

- Answer question one (**Compulsory**) and any other **Two** questions.
- Attempted questions must be indicated on front cover of answer booklet.
- Every question should be started on new page and question indicated respectively.

Question One (30 Marks)

a. State the principle of conservation of energy (2mks)

b. State Newton's second law of motion. (2mks)

c. Two skaters collide and embrace each other. One has mass m_1 = 70kg and is initially moving east at a speed u_1= 8km/h while the other has mass m_2 = 50kg and is initially moving north at a speed u_2 = 10km/h.

   i. What is the final velocity of the couple? (3mks)

   ii. What fraction of the initial kinetic energy is lost because of the collision? (3mks)

d. Discuss Holonomic Constraints and non-holonomic Constraints giving an example in each case. (4mks)

e. Define centre of mass of a system of particles (2mks)

f. Distinguish between inertia mass and gravitational mass of an accelerated object. (3mks)

g. In a radioactive decay, an alpha particle, which is the nucleus of the He-atom, is emitted from a Uranium-238 nucleus originally at rest, with a speed of $1.4 \times 10^7 ms^{-1}$ and a kinetic energy of 4.1MeV. Find the recoil speed of the residual nucleus which is Thorium-234. (4mks)

h. Discuss the difference between kinematics and dynamics as used in study of motion and state the limitation of classical mechanics. (3mks)

i. A rocket is moving away from the solar system at a speed of $6.0 \times 10^3 ms^{-1}$. It fires its engine which ejects exhaust with a relative velocity of $3.0 \times 10^3 ms^{-1}$. The mass of the rocket at this time is 4.0×10^4kg and it experiences an acceleration of $2.0ms^{-2}$. Fin

   i. The velocity of the exhaust relative to the solar system. (3mks)

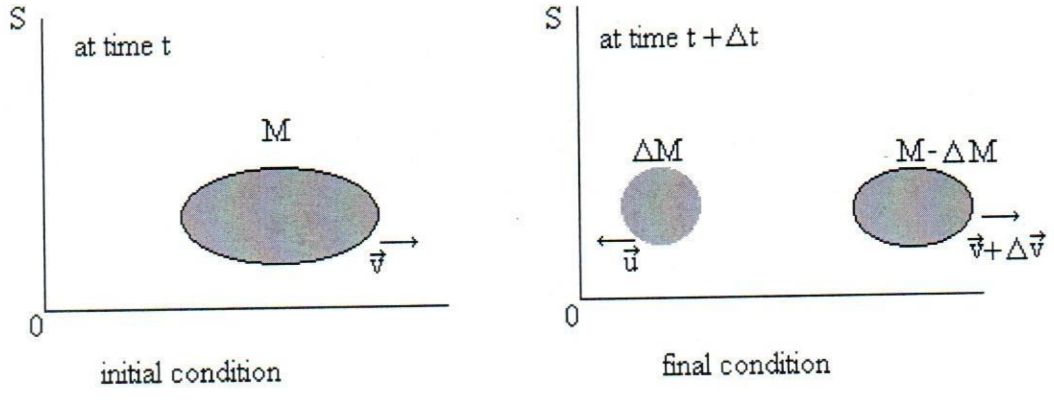
   ii. The rate at which the exhaust was ejected during the firing. (3mks)

Question Two (20 Marks)

a. The position vector of a particle v and its velocity relative to the centre of mass are $\vec{r}_v'$ and $\vec{v}_v'$ respectively. Prove that

(i) $\sum_v m_v \vec{r}_v' = 0$ and (ii) $\sum_v m_v \vec{v}_v' = 0$ (6 marksks)

b. Given that the initial mass of the rocket and the fuel it contains as M and its center of mass velocity as seen from some frame of reference S after time Δt as $\vec{v}$, as shown in fig.(a).



(a) (b)

After change in time Δt, the mass ΔM has been ejected in the form of gases generated within the rocket as that much amount of fuel has been burnt during this time. Letting $\vec{u}$ be the velocity of the center of mass of the ejected gas and $\vec{v}+\Delta\vec{v}$ be velocity of the center of mass of the rocket and the remaining fuel system as observed from the given frame of reference as shown in fig. (b). Show that Newton second law of motion of variable mass is given by

$$\vec{F}_{ext} = \frac{d}{dt}(M\vec{v}) - \vec{u}\frac{dM}{dt}$$ (8Marks)

c. Show that for a single particle with constant mass the equation of motion implies the following differential equation for the kinetic energy: $\frac{dT}{dt} = F.v$ while if the mass varies with time the corresponding equation is $\frac{d(mT)}{dt} = F.p$ (6marks)

Question Three (20 Marks)

a. Use Hamilton's Principle to find the equation of motion of a one-dimensional harmonic oscillator. (8 Marks)

b. An object is projected vertically upward from the earth's surface with initial speed v_o. Neglecting air resistance, show that the speed of the object at a distance H above the earth's surface is

$$v = \sqrt{v_o^{\ 2} - \frac{2GMH}{R(R+H)}}$$ (6 Marks)

c. Given that the mass m is attached to a spring of force constant k obtain its equation of motion and frequency of oscillation using Lagrange equations. (6marks)

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Question four (20 Marks)

a. Rockets are propelled by the momentum reaction of the exhaust gases expelled from the tail. Since these gases arise from the reaction of the fuels carried in the rocket the mass of the rocket is not constant, but decreases as the fuel is expended. Show that the equation of motion for a rocket projected vertically upward in a uniform gravitational field, neglecting atmospheric resistance, is

$$m\frac{dv}{dt} = v'\frac{dm}{dt} - mg$$

Where m is the mass of the rocket and $v'$is the velocity of the escaping gases relative to the rocket. Integrate this equation to obtain v as a function of m, assuming a constant time rate of loss of mass. Show, for a rocket starting initially from rest, with v' equal to 6800 ft/sec and a mass loss per second equal to $\frac{1}{60}th$ of the initial mass, that in order to reach the escape velocity the ratio of the weight of the fuel to the weight of the empty rocket must be almost 300. (8 marks)

b. Consider a circular disc of mass *M* and radius *R* rolling down an inclined plane without slipping. Find the speed of its center of mass when it reaches the bottom of the incline. (6 marks)

c. Prove that the magnitude R of the position vector for the center of mass from an arbitrary origin is given by the equation

$$M^2R^2 = M\sum_i m_i r_i^2 - \frac{1}{2}\sum_{ij} m_i m_j r_{ij}^2 \qquad \text{(6 marks)}$$