



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR
FIRST YEAR FIRST SEMESTER
MAIN EXAMINATION

**FOR THE DEGREES OF BACHELOR OF EDUCATION AND
BACHELOR OF SCIENCE**

COURSE CODE: STA 112/STA 142

COURSE TITLE: INTRODUCTION TO PROBABILITY

DATE: 14/12/2022

TIME: 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 5 Printed Pages. Please Turn Over.

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF
SCIENCE(MATHEMATICS)

CODE: STA 112

TITLE: INTRODUCTION TO PROBABILITY

TIME: 2 HOURS

DATE: DECEMBER, 2022

QUESTION ONE (30 MARKS)

1. (a) Define the following terms: A set, Equally likely events, Sample (3 mks)
- (b) Let A and B be any two events defined on a sample space S. State any three probability conditions for events A and B. (3 mks)
- (c) A fair coin was tossed twice. Let X be the number of heads that turned up. Find the probability function of X. (3 mks)
- (d) i. Explain the following: discrete random variable and continuous random variable (2 mks)
ii. A random variable X has the probability distribution below

X	0	1	2	3	4	5
P(X=x)	a	3a	5a	7a	9a	11a

- A. Determine the value of a (2 mks)
- B. Find $P(X < 2)$, $P(0 < X < 3)$ (4 mks)
- (e) A committee of 4 people need to be selected from 5 women and 7 men. How many ways can the committee be selected if at least 3 women must be included. (4 mks)
- (f) Let $A = \{9, 2, 1\}$ and $B = \{3, 5, 6\}$. Show that A and B are commutative. (3 mks)
- (g) Five lifeguards are available for duty one Saturday afternoon. There are three lifeguard stations. In how many ways can three lifeguards be chosen and ordered among the stations? (3 mks)
- (h) If A and B be two independent events in S, show that A and B' are also independent. (3 mks)

QUESTION TWO (20 MARKS)

2. (a) Let X be random variable with pdf

$$f(x) = \begin{cases} \frac{1}{6}x, & x = 0, 1, 2, 3 \\ 0, & \text{elsewhere} \end{cases}$$

Compute;

- i. $E(X)$, (2 mks)
 - ii. $2E(X)$ (2 mks)
 - iii. $E(X^2)$ (2 mks)
 - iv. $Var(X)$
- (b) Suppose A and B be two events defined on a sample space S such that $P(A) = 0.3$, $P(B) = 0.5$, and $P(A \cup B) = 0.7$. Find;
- i. $P(A \cap B)$ (2 mks)
 - ii. $P(A^c \cup B^c)$ (2 mks)
 - iii. $P(A^c \cap B)$
- (c) The proportion of people in a given community who have a certain disease is 0.005. A test is available to diagnose the disease. If a person has the disease, the probability that the test will produce a positive signal is 0.99. If a person does not have the disease, the probability that the test will produce a positive signal is 0.01. If a person tests positive, what is the probability that the person actually has the disease? Explain why a large proportion of those who test positive are actually disease free. (6 mks)

QUESTION THREE (20 MARKS)

3. (a) Three presidential candidates A , B and C were interviewed on a national TV station. From the interview, it is estimated that 30% of the population support A , 26% support B and 24% support C , 8% support A and B , 5% support A and C , 4% support B and C , and 2% support all the three candidates. Represent this information on a Venn diagram and find the probability that a randomly chosen person:

- i. do not support any candidate (2 mks)
 - ii. supports A but not B (2 mks)
 - iii. does not support C (2 mks)
 - iv. supports only one of these candidates (2 mks)
 - v. supports only two of these candidates (2 mks)
- (b) Let X be a random variable with the probability distribution function

$$f(x) = \begin{cases} \frac{1}{2}(x+1), & -1 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find

- i. $E(X)$ (3 mks)
- ii. $Var(X)$ (5 mks)
- iii. $Var(5X + 10)$ (2 mks)

QUESTION FOUR (20 MARKS)

4. (a) Consider tossing two fair dice. Let X denote the sum of the upturned values of the two dice and Y their absolute difference. Calculate the expected value of X and Y . (8 mks)
- (b) Let X be a random variable representing the quantity of sugar (in tonnes) sold on a day at a certain factory with a distribution function as shown;

$$f(x) = \begin{cases} Kx, & 0 \leq x \leq 5 \\ K(10 - x), & 5 < x \leq 10 \\ 0, & \text{elsewhere} \end{cases}$$

- i. Find K such that $f(x)$ is a pdf (4 mks)
- ii. Find $P(X \leq 5)$ (2 mks)
- iii. Find $P(X > 5)$ (2 mks)
- iv. Find $P(2.5 \leq X \leq 7.5)$ (4 mks)

QUESTION FIVE (20 MARKS)

5. (a) Two urns each contain three cards. The first urn contains beads labeled 1, 3 and 5. The second urn contains cards labeled 2, 6, and 8. In a game, a player draws one card at random from each urn and his score, X , is the sum of the numbers on the two beads.
- i. Obtain the six possible values of X and find their corresponding probabilities (2 mks)
- ii. Calculate the standard deviation of X . (8 mks)
- (b) A six-sided die has faces marked with the numbers 1, 3, 5, 7, 9, and 11, it is biased so that the probability of obtaining the number R in a single roll of the die is proportional to R .
- i. Show that the probability distribution of R is given by $P(R = r) = \frac{r}{36}$, $r = 1, 3, 5, 7, 9, 11$ (3 mks)
- ii. The die is to be rolled and a rectangle drawn with sides of lengths 6 cm and R cm. calculate the expected value of the area of the rectangle (4 mks)
- iii. The die is to be rolled again and a square drawn with sides of length $24R^{-1}$ cm. Calculate the expected value of the perimeter of the square (3 mks)