(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREES OF BACHELOR OF SCIENCE

COURSE CODE: STA 112

COURSE TITLE: INTRODUCTION TO PROBABILITY

DATE: ~~21~~ 14/12/22 **TIME: 2 PM – 4PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 printed pages. Please Turn Over.

QUESTION ONE (30 MARKS)

1. (a) Define the following terms: A set, Equally likely events, Sample (3 mks)

   (b) Let A and B be any two events defined on a sample space S. State any three probability conditions for events A and B. (3 mks)

   (c) A fair coin was tossed twice. Let X be the number of heads that turned up. Find the probability function of X. (3 mks)

   (d) i. Explain the following: discrete random variable and continuous random variable (2 mks)

   ii. A random variable X has the probability distribution below

X	0	1	2	3	4	5
P(X=x)	a	3a	5a	7a	9a	11a

   A. Determine the value of a (2 mks)

   B. Find $P(X < 2)$, $P(0 < X < 3)$ (4 mks)

   (e) A committee of 4 people need to be selected from 5 women and 7 men. How many ways can the committee be selected if at least 3 women must be included. (4 mks)

   (f) Let $A = \{9, 2, 1\}$ and $B = \{3, 5, 6\}$. Show that A and B are commutative. (3 mks)

   (g) Five lifeguards are available for duty one Saturday afternoon. There are three lifeguard stations. In how many ways can three lifeguards be chosen and ordered among the stations? (3 mks)

   (h) If A and B be two independent events in S, show that A and B' are also independent. (3 mks)

QUESTION TWO (20 MARKS)

2. (a) Let X be random variable with pdf

$$f(x) = \begin{cases} \frac{1}{6}x, & x = 0, 1, 2, 3 \\ 0, & elsewhere \end{cases}$$

Compute;

i. $E(X)$, (2 mks)

ii. $2E(X)$ (2 mks)

iii. $E(X^2)$ (2 mks)

iv. $Var(X)$ (2 mks)

(b) Suppose A and B be two events defined on a sample space S such that $P(A) = 0.3$, $P(B) = 0.5$, and $P(A \bigcup B) = 0.7$. Find;

i. $P(A \bigcap B)$ (2 mks)

ii. $P(A^c \bigcup B^c)$ (2 mks)

iii. $P(A^c \bigcap B)$ (2 mks)

(c) The proportion of people in a given community who have a certain disease is 0.005. A test is available to diagnose the disease. If a person has the disease, the probability that the test will produce a positive signal is 0.99. If a person does not have the disease, the probability that the test will produce a positive signal is 0.01. If a person tests positive, what is the probability that the person actually has the disease? Explain why a large proportion of those who test positive are actually disease free. (6 mks)

QUESTION THREE (20 MARKS)

3. (a) Three presidential candidates A, B and C were interviewed on a national TV station. From the interview, it is estimated that 30% of the population support A, 26% support B and 24% support C, 8% support A and B, 5% support A and C, 4% support B and C, and 2% support all the three candidates. Represent this information on a Venn diagram and find the probability that a randomly chosen person:

i. do not support any candidate (2 mks)

ii. supports A but not B (2 mks)

iii. does not support C (2 mks)

iv. supports only one of these candidates (2 mks)

v. supports only two of these candidates (2 mks)

(b) Let X be a random variable with the probability distribution function

$$f(x) = \begin{cases} \frac{1}{2}(x + 1), & -1 < x < 1 \\ 0, & elsewhere \end{cases}$$

Find

i. $E(X)$ (3 mks)

ii. $Var(X)$ (5 mks)

iii. $Var(5X + 10)$ (2 mks)

QUESTION FOUR (20 MARKS)

4. (a) Consider tossing two fair dice. Let X denote the sum of the upturned values of the two dice and Y their absolute difference. Calculate the expected value of X and Y. (8 mks)

   (b) Let X be a random variable representing the quantity of sugar (in tonnes) sold on a day at a certain factory with a distribution function as shown;

$$f(x) = \begin{cases} Kx, & 0 \leq x \leq 5 \\ K(10-x), & 5 < x \leq 10 \\ 0, & elsewhere \end{cases}$$

   i. Find K such that $f(x)$ is a pdf (4 mks)
   ii. Find $P(X \leq 5)$ (2 mks)
   iii. Find $P(X > 5)$ (2 mks)
   iv. Find $P(2.5 \leq X \leq 7.5)$ (4 mks)

QUESTION FIVE (20 MARKS)

5. (a) Two urns each contain three cards. The first urn contains beads labeled 1, 3 and 5. The second urn contains cards labeled 2, 6, and 8. In a game, a player draws one card at random from each urn and his score, X, is the sum of the numbers on the two beads.

   i. Obtain the six possible values of X and find their corresponding probabilities (2 mks)
   ii. Calculate the standard deviation of X. (8 mks)

   (b) A six-sided die has faces marked with the numbers 1,3,5,7,9, and 11, it is biased so that the probability of obtaining the number R in a single roll of the die is proportional to R.

   i. Show that the probability distribution of R is given by $P(R = r) = \frac{r}{36}$, $r = 1, 3, 5, 7, 9, 11$ (3 mks)
   ii. The die is to be rolled and a rectangle drawn with sides of lengths 6 cm and R cm. calculate the expected value of the area of the rectangle (4 mks)
   iii. The die is to be rolled again and a square drawn with sides of length $24R^{-1}$cm. Calculate the expected value of the perimeter of the square (3 mks)