(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE BACHELOR OF SCIENCE

COURSE CODE: **MAT 351**

COURSE TITLE: **ENGINEERING MATHEMATICS III**

DATE: 19/12/2022 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any Other TWO Questions
TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms as used in engineering mathematics. (2 Marks)
   i) Engineering Mathematics
   ii) Fourier Series

b) State one importance of Laplace Transform. (1 Marks)

c) Write down the solutions of the following Laplace transform formulas giving relevant conditions for each. (9Marks)

   (i) $L(1)$ (ii) $L(t^n)$ (iii) $t^n e^{at}$ (iv) $L[Cosh(at)]$

d) Given $F(s)=\frac{2}{s^5}$ find f (t). (5 Marks)

e) Prove that $L[af_1(t)+bf_2(t)]=aL[f_1(t)]+bL[f_2(t)]$ (3 Marks)

f) Write down the solutions for the following inverse Laplace transforms.

   i) $L^{-1}\left(\frac{1}{s}\right)$ (ii) $L^{-1}\left(\frac{1}{s^n}\right)$ (iii) $L^{-1}\left(\frac{1}{s-a}\right)$ (iv) $L^{-1}\left(\frac{s}{s^2-a^2}\right)$ (4 Marks)

g) Derive a divergence of a vector function. (5 Marks)

h) State Green's Theorem. (1 Mark)

QUESTION TWO (20 MARKS)

a) Use Laplace transforms to solve $\frac{dy}{dt}=3-2t$ with y(0)=0 $y^1(0)=0$

(5 marks)

b) Expand $f(x)=x^2$ $0\angle x\angle L$ in Fourier cosine series and Fourier sine series.

(10 marks)

c) Evaluate the value of $\lim_{x\to 0}\frac{\sin x}{x}$ (5 marks)

QUESTION THREE (20 MARKS)

a) State four Dirichlet's conditions for a Fourier series. (4Marks)

b) If $z(x+y)=x^2+y^2$ show that

$$\left[\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}\right]^2=4\left[1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right]$$

(10 marks)

c) Differentiate between an even and an odd function giving two example for each case (6marks)

QUESTION FOUR 20 MARKS

a) With the aid of standard tables evaluate

$$L^{-1}\left\{\frac{3s+5}{s^2+7}\right\}$$ (6 marks)

b) Evaluate $\lim_{x \to 2} \frac{x^3-8}{x-2}$ (5 marks)

c) Find the unit normal to the surface $xy^3z^2 = 4\ at\ (-1,-1,2)$ (5 marks)

d) Calculate $\nabla.r \quad ;r = xi + yj + zk$ (4Marks)

QUESTION FIVE (20 MARKS)

Using the Laplace transforms find the solution to the initial value problem

$y^{11} - 2y^{1} - 8y = 0 \quad y(0) = 3 \;\; y^{1}(0) = 6$ (20 marks)

Table of Laplace Transforms

$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$	$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}$	2. e^{at}	$\frac{1}{s-a}$
3. t^n, $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4. t^p, $p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5. $\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6. $t^{n-\frac{1}{2}}$, $n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7. $\sin(at)$	$\frac{a}{s^2+a^2}$	8. $\cos(at)$	$\frac{s}{s^2+a^2}$
9. $t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10. $t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11. $\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12. $\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13. $\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14. $\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15. $\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16. $\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17. $\sinh(at)$	$\frac{a}{s^2-a^2}$	18. $\cosh(at)$	$\frac{s}{s^2-a^2}$
19. $e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20. $e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21. $e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22. $e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23. $t^n e^{at}$, $n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25. $u_c(t)=u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26. $\delta(t-c)$ Dirac Delta Function	e^{-cs}
27. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	28. $u_c(t)g(t)$	$e^{-cs}\mathcal{L}\{g(t+c)\}$
29. $e^{ct}f(t)$	$F(s-c)$	30. $t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31. $\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32. $\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34. $f(t+T)=f(t)$	$\frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}}$
35. $f'(t)$	$sF(s)-f(0)$	36. $f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37. $f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$		