(Knowledge for Development)

KIBABII UNIVERSITY

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UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: STA 112/STA 142

COURSE TITLE: INTRODUCTION TO PROBABILITY

DATE: 04/02/2022   **TIME:** 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

1. (a) Define the following terms: Sample outcome, finite set, exhaustive events (3 mks)

   (b) For two events A and B, suppose that $P(A) = 0.3$, $P(B) = 0.5$, and $P(A \cup B) = 0.6$. Calculate $P(A \cap B)$. (3 mks)

   (c) Let A be the set of x's for which $x^2 + 2x = 8$; let B be the set for which $x^2 + x = 6$. Find $A \cap B$ and $A \cup B$ (4 mks)

   (d)   i. Explain the following: discrete random variable and continuous random variable (2 mks)

         ii. A random variable X has the probability distribution below

X	0	1	2	3	4	5	6	7	8
P(X=x)	a	2a	3a	4a	5a	6a	7a	8a	9a

         A. Determine the value of a (2 mks)
         B. Find $P(X < 3)$, $P(0 < X < 5)$ (4 mks)

   (e) A committee of 4 people need to be selected from 5 women and 7 men. How many ways can the committee be selected if at least 3 women must be included. (3 mks)

   (f) A fair die is thrown twice. A is the event "sum of the throws equals 4," B is "at least one of the throws is a 3." Calculate $P(A|B)$; Are A and B independent? (3 mks)

   (g) If two fair dice are rolled once, what is the probability that the total number of spots shown is equal to 5? (3 mks)

   (h) At a party with 10 guests, every guest shakes hands with each other. How many handshakes can be counted in total? (3 mks)

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QUESTION TWO (20 MARKS)

2. (a) Let X be random variable with pdf

$$f(x) = \begin{cases} \frac{x}{5}, & x = 1, 2, 3, 4 \\ 0, & elsewhere \end{cases}$$

Compute;
                                                          (2 mks)

    i. $E(X)$,   (3 mks)

    ii. $E(3X)$ (3 mks)

    iii. $Var(X)$

(b) Kibabii university library subscribes to three news magazines A, B, and C according to the following proportions: A : 20%, B : 15%, C : 10%, both A and B : 5%, both A and C : 4%, both B and C : 3%, all three A, B, and C : 2%. If a student is chosen at random, what is the probability he/she subscribes to none of the news magazines? (6 mks)

(c) Let $A, B \in S$, show that
                                                           (3 mks)

    i. $P(A) = P(A) - P(A \cap B)$ (3 mks)

    ii. $P(A') = 1 - P(A)$

QUESTION THREE (20 MARKS)

3. (a) A university library has five copies of a textbook to be used in a certain class. Of these copies, numbers 1 through 3 are of the 1st edition, and numbers 4 and 5 are of the 2nd edition. Two of these copies are chosen at random to be placed on a 2-hour reserve.
                                                                    (2 mks)

    i. Write out an appropriate sample space S.

    ii. Consider the events A, B, C, and D, defined as follows, and express them in terms of sample points.
          A = both books are of the 1st edition,   (2 mks)
          B = both books are of the 2nd edition,   (2 mks)
          C = one book of each edition,   (2 mks)
          D = no book is of the 2nd edition.   (2 mks)

(b) Let X have the pdf

$$f(x) = \begin{cases} \frac{1}{2}(x+1), & -1 < x < 1 \\ 0, & elsewhere \end{cases}$$

Find
                                                            (5 mks)

    i. $Var(X)$ (5 mks)

    ii. $Var(5X)$

QUESTION FOUR (20 MARKS)

4. (a) Consider tossing two fair dice. Let X denote the sum of the upturned values of the two dice and Y their absolute difference. Calculate the expected value of X and Y. (8 mks)

(b) Let X (in tonnes) be a random variable representing the quantity of sugar sold in a day at a certain factory with a distribution function as shown;

$$f(x) = \begin{cases} cx, & 0 \leq x \leq 3 \\ c(10 - x), & 3 < x \leq 5 \\ 0, & elsewhere \end{cases}$$

  i. Find c such that $f(x)$ is a pdf   (4 mks)

  ii. Find $P(X \leq 3)$ (2 mks)

  iii. Find $P(X > 3)$ (2 mks)

  iv. Find $P(2.5 \leq X \leq 5)$ (4 mks)

QUESTION FIVE (20 MARKS)

5. (a) Two boxes each contain three cards. The first box contains cards labeled 1, 3 and 5. The second box contains cards labeled 1, 3, and 5. In a game, a player draws one card at random from each box and his score, X, is the sum of the numbers on the two cards.

  i. Obtain the six possible values of X and find their corresponding probabilities   (2 mks)

  ii. Calculate the standard deviation of X.   (6 mks)

(b) A driving licence examination consists of two parts which are based on a theoretical and a practical examination. Suppose 25% of people fail the practical examination, 15 % of people fail the theoretical examination, and 10 % of people fail both the examinations. If a person is randomly chosen, then what is the probability that this person

  i. fails at least one of the examinations?   (3 mks)

  ii. only fails the practical examination, but not the theoretical examination? (3 mks)

  iii. successfully passes both the tests?   (3 mks)

  iv. fails any of the two examinations?   (3 mks)