(KNOWLEDGE FOR DEVELOPMENT)

KIBABII UNIVERSITY
(KIBU)

UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR

SPECIAL/SUPPLEMENTARY EXAMINATIONS
FIRST YEAR SEMESTER TWO

FOR THE DEGREE IN
(COMPUTER SCIENCE)

COURSE CODE: CSC 121

COURSE TITLE: DISCRETE STRUCTURES II

DATE: 30/09 / 2021 TIME: 02.00 P.M – 04.00 P.M

INSTRUCTIONS

ANSWER QUESTIONS ONE AND ANY OTHER TWO

QUESTION ONE [COMPULSORY] 30 MARKS]

a. Studying logic means study proof. Justify this claim. **[2 marks]**

b. When is it appropriate to apply deduction process and not computation process? At what point to these two strategies converge? **[4 marks]**

c. Using relevant statements and formalization differentiate between Modus Pones and Modus Tollens. **[4 marks]**

d. Use the Euclidean algorithm to find the greatest common divisor of 46 and 21. Hence or otherwise find integers s and t satisfying that $\gcd(46, 21) = s \cdot 46 + t \cdot = 11$ **[4 marks]**

e. Determine all integers x such that $x \equiv 2 \pmod{46}$ and $x \equiv 1 \pmod{21}$. **[3 marks]**

f. Write the following statement in symbolic form using quantifiers:

(i) All students have taken a course in mathematics **[2 marks]**

(ii) Some students are intelligent, but not hardworking **[2 marks]**

g. What is the probability that when two dice are rolled, the sum of the numbers on the two dice is more than 8? **[3 marks]**

h. Find the coefficient of x^3y^4 in the binomial expansion of $(2x-3y)^6$. **[3 marks]**

i. Consider the sequence given recursively by $a_1=4$, $a_2=6$ and $a_n=a_{n-1} + n_{a-2}$. Write out the first 6 terms of the sequence. **[3 marks]**

QUESTION TWO [20 MARKS]

a. Find the recurrence relation with initial condition for the following: 2, 10, 50, 250, . . . **[2 marks]**

b. Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} - a_{n-2}$ for n =2 , 3 , 4, . . . , and suppose that $a_o = 3$ and $a_l = 5$. What are a_2 and a_3? **[2 marks]**

c. Determine the generating function of the following numeric function $a_n = 2^n$, if n is even **[2 marks]**

d. Use mathematical induction to show that $2 + 4 + 6 + \ldots + 2n = n^2 + n$, for $n \geq 1$ **[4 marks]**

e. Given two integers **a** and **b** such that a=2740 and b=1760 using extended Euclidian algorithm find the values of integers **s** and **t** such that **s(a)+t(b)=Gcd(a,b)** **[4 marks]**

f. Find a positive integer *(x)* such that when *(x)* is divided by **5** it gives a remainder of **4**, when divided by **7** remainder is **6** and when divided by **9** remainder is **8**. **[6 marks]**

QUESTION THREE [20 MARKS]

a. Define the following terms as used in the study of discrete structures

- **i.** Linear Congruence **[1 mark]**
- **ii.** Equiprobable Spaces **[1 mark]**
- **iii.** Random Variables **[1 mark]**
- **iv.** Independent Event **[1 mark]**

b. A pair of dice is loaded. The probability that a 4 appears on the first die is 2/7, and the probability that a 3 appears on the second die is 2/7. Other outcomes for each die appear with probability 1 /7. What is the probability of 7 appearing as the sum of the numbers when the two dice are rolled? **[3 marks]**

c. What is the probability that the numbers 11,4, 17,39, and 23 are drawn in that order from a bin containing 50 balls labeled with numbers 1,2, . . ., 50 if:

- i. The ball selected is not returned to the bin before the next ball is selected and; **[3 marks]**
- ii. The ball selected is returned to the bin before the next ball is selected? **[3 marks]**

d. There are many gambling games nowadays that award enormous prizes to people who correctly choose a set of six numbers out of the first n positive integers where n is usually between 30 and 60. What is the probability that a person picks the correct six numbers out of 40? **[3 marks]**

e. Prove using counting argument that C (n, r) = C (n-1, r) + C (n-1, r-1). **[4 marks]**

QUESTION FOUR [20 MARKS]

a. Differentiate between a Graph and a Tree and a spanning tree with an example in each case. **[4 marks]**

b. Suppose that a graph G(V,E) is such that V_1 is of degree 1, V_2 is of degree 2, V_3 vertices of degree 3 and V_5 vertices of degree 4. Can such a graph exist? **[3 marks]**

c. Given that the graph K_n has 21 edges.

i. Find the number of vertices that K_n is composed of. **[2 marks]**

ii. Determine the degree of each vertex. **[2 marks]**

iii. Calculate the sum of the degrees of all its vertices. **[2 marks]**

d. Graph G is represented by the following adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

i. Draw the graph G. **[3 marks]**

ii. Determine whether G is a tree. Justify your answer. **[2 marks]**

iii. Determine whether G is Eulerian graph. Justify your answer.

[2 marks]

QUESTION FIVE [20 MARKS]

a. Define the expected value (or expectation) of the random variable X (s) on the sample space S. Suppose X is the number that comes up when a die is rolled. What is the expected value of X? **[4 marks]**

b. State the generalized pigeon hole principle and show that, among 100 people, at least 9 of them were born in the same month. **[4 marks]**

c. Consider the convention used in assigning a number plate to vehicles in Kenya, where a valid number plate has two parts. The first part consists of Three (3) Alphabetical Letters of which the first letter is **K** followed by two letters orderly selected from [A-Z] {except I and O. the second part consist of three (3) Numbers orderly selected from [0-9] and a Letter [A-Z] {except I and O}.

Example **KDC 671Q**

How many number plates will be issued before abolishing this convention? **[4 marks]**

d. Differentiate between N-P hard and NP-complete problem. **[2 marks]**

e. Arrange the following expressions by growth rate from slowest to the fastest and state the value of n of which expression will be most efficient.

$4n^2$,	**$\log_3 n$,**	**n! ,**	**3^n,**	**$2 \log_2 n$**

[6 marks]