# KIBABII UNIVERSITY

## UNIVERSITY EXAMINATIONS

## 2022/2023 ACADEMIC YEAR

### FIRST SEMESTER
### MAIN EXAMINATIONS

FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

**COURSE CODE:** SPH 811

**COURSE TITLE:** MATHEMATICAL PHYSICS

DURATION: 3 HOURS

**DATE: 14/12/2022** **TIME: 9:00-11:00AM**

---

INSTRUCTIONS TO CANDIDATES

- Answer **ANY THREE QUESTIONS**.
- Each question carries **20 MARKS**.
- ALL Symbols have their usual meaning
- $\int_0^{\infty} r^{-1} e^{-r^2} d r = 0$

## QUESTION ONE (20 MARKS)

a) Show that $\vec{\nabla} r^n = nr^{n-2}\vec{r}$. (5marks)

b) A vector field is given by $\vec{A} = (x_1 + 2x_2 + ax_3)\hat{e}_1 + (bx_1 - 3x_2 - x_3)\hat{e}_2 + (4x_1 + cx_2 + 2x_3)\hat{e}_3$. Find the constants a, b and c such that the vector field is irrotational. (3marks)

c) Prove that $\vec{\nabla} \times (\vec{\nabla}\emptyset) = \mathbf{0}$. (2marks)

d) Show that $B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$. (4marks)

e) Evaluate using the properties of beta and gamma function the integral $\int_0^1 x^5 (1-x)^4 dx$. (2marks)

f) Use the Stokes vector integral theorem to verify the Maxwell's equation of electromagnetism i.e. $\vec{\nabla} \times \vec{B} = \mu_o \vec{J}$. (4marks)

## QUESTION TWO (20 MARKS)

a) Given $\vec{V}_1 = (1,1,1,1)$, $\vec{V}_2 = (1,1,1,0)$, $\vec{V}_3 = (1,1,0,0)$ and $\vec{V}_4 = (1,0,0,0)$ is a basis of $\mathbb{R}^4$, construct by using G-S procedure an orthonormal basis for $\mathbb{R}^4$. (8marks)

b) Determine the eigen values and the corresponding eigen vectors of the matrix.
$$\begin{pmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{pmatrix}$$ (8marks)

c) Find the inverse Laplace transform of $F(s) = \frac{2-5s}{(s-6)(s^2+11)}$. (4marks)

## QUESTION THREE (20 MARKS)

a) Evaluate by Cauchy's integral formula $\oint \frac{e^{2z}}{(z+1)^4} dz$ where C is any simple closed curve for the cases,
   i. C does not enclose $z = -1$. (2marks)
   ii. C encloses the point $z = -1$. (4marks)

b) Use the gamma function to evaluate $\Gamma\left(\frac{1}{2}\right)$. (9marks)

c) Use the result in (a) above to evaluate $\int_0^\infty x^{\frac{1}{2}} e^{-x^2} dx$. (5marks)

## QUESTION FOUR (20 MARKS)

a) Use the calculus of residues to show that $\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$ where $a > b > 0$ (5marks)

b) Obtain the first and second forms of the Greens Theorem. (5marks)

c) Solve the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3t}$ $given\ y(0) = 1\ and\ y'(0) = 0$ (5marks)

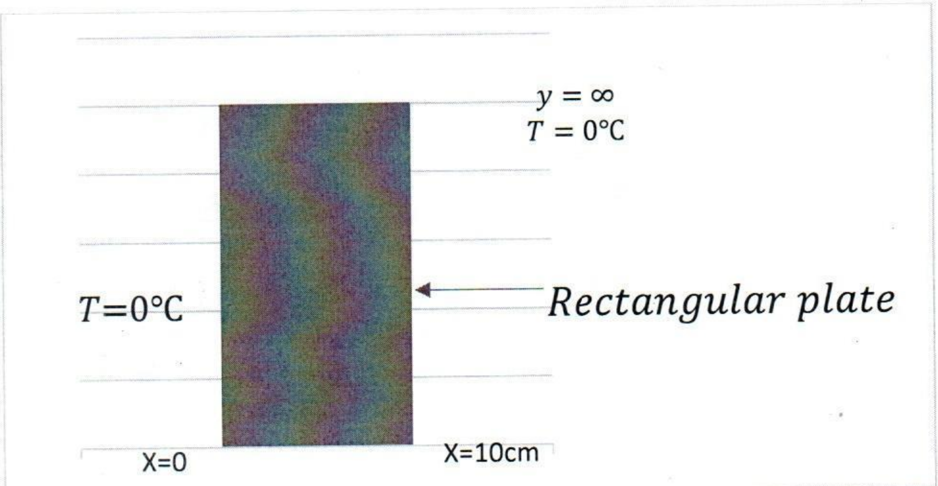
d) The graph of an electrical signal is given by $\boldsymbol{i = 5\sin\frac{\theta}{2}\ for\ 0 \leq \theta \leq 2\pi}$. Obtain the Fourier series that would represent this alternating current. (5marks)

## QUESTION FIVE (20 MARKS)

a) The equation of a wave is given as $\boldsymbol{\nabla^2 u = \frac{1}{v^2}\frac{\partial^2 y}{\partial x^2}}$. Use the method of separation of variables to find the amplitude at any given time of a standing wave produced on a string. (6marks)

b) Using the Schrodinger equation derive the ground state wave function for a free particle in a one-dimensional case. (6marks)

c) A long rectangular plate has its long sides and the far end at $0^{\circ}C$ and the base at $100^{\circ}C$. The width of the plate is 10cm. Find the steady state temperature inside the plate. (8marks)




e) Given that $\vec{V_1} = (2,-1,0)$ , $\vec{V_2} = (1,0,-1)$ and $\vec{V_3} = (3,7,-1)$ is a basis of $R^3$. Find the orthogonal basis by Gram-Schmidt procedure hence determine the orthonormal basis (6marks)

## Table 15.2 Laplace Transforms

| $f(s)$ | $F(t)$ | Limitation |
|---|---|---|
| 1. $1$ | $\delta(t)$ | Singularity at $+0$ |
| 2. $\dfrac{1}{s}$ | $1$ | $s > 0$ |
| 3. $\dfrac{n!}{s^{n+1}}$ | $t^n$ | $s > 0$<br>$n > -1$ |
| 4. $\dfrac{1}{s-k}$ | $e^{kt}$ | $s > k$ |
| 5. $\dfrac{1}{(s-k)^2}$ | $te^{kt}$ | $s > k$ |
| 6. $\dfrac{s}{s^2-k^2}$ | $\cosh kt$ | $s > k$ |
| 7. $\dfrac{k}{s^2-k^2}$ | $\sinh kt$ | $s > k$ |
| 8. $\dfrac{s}{s^2+k^2}$ | $\cos kt$ | $s > 0$ |
| 9. $\dfrac{k}{s^2+k^2}$ | $\sin kt$ | $s > 0$ |
| 10. $\dfrac{s-a}{(s-a)^2+k^2}$ | $e^{at}\cos kt$ | $s > a$ |
| 11. $\dfrac{k}{(s-a)^2+k^2}$ | $e^{at}\sin kt$ | $s > a$ |
| 12. $\dfrac{s^2-k^2}{(s^2+k^2)^2}$ | $t\cos kt$ | $s > 0$ |
| 13. $\dfrac{2ks}{(s^2+k^2)^2}$ | $t\sin kt$ | $s > 0$ |