KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR

FIRST SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

COURSE CODE: SPH 813

COURSE TITLE: ELECTRODYNAMICS

DURATION: 3 HOURS

DATE: 14/12/2022 **TIME: 2:00-4:00PM**

INSTRUCTIONS TO CANDIDATES

- Answer **ANY THREE QUESTIONS.**
- Each question carries **20 MARKS.**

- $\mu_o = 4\pi \times 10^{-7} T.m/A$, $\mu_o = \frac{4\pi}{c}$

$$\vec{\nabla} \times \vec{B} \equiv \left(\frac{1}{r}\frac{\partial B_z}{\partial \varphi} - \frac{\partial B_\varphi}{\partial z}\right)\hat{r} + \left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right)\hat{\varphi} + \frac{1}{r}\left(\frac{\partial (rB_\varphi)}{\partial r} - \frac{\partial B_r}{\partial \varphi}\right)\hat{z}.$$

$$\vec{\nabla} \circ \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

QUESTION ONE (20 MARKS)

a) Use Maxwell's equations to express the electric field $\boldsymbol{B}$ in terms of the general wave equation. (5marks)

b) Show that the electric field of an electrically neutral system is given by $\boldsymbol{E} = \frac{3r(r.D)-r^2D}{r^5}$

Where symbols have their usual meaning. (6marks)

a) Use Biot-Savart law to show that the divergence of all magnetic fields is zero. (5marks)

c) The expression for the magnetic field of a circular parallel plate capacitor is $\vec{\boldsymbol{B}} = \frac{\mu_0 \Delta r}{2\pi R} e^{-\frac{t}{RC}} \left(\frac{r}{r_o^2}\right) \hat{\boldsymbol{\varphi}}$. Use this result to find the displacement current density between the plates. (4marks)

QUESTION TWO (20 MARKS)

a) Derive the components of the electric field for a moving charged particle in matter in both fixed and moving reference frames. (5marks)

b) Show that the varying electric field is rotational whereas the electrostatic field is irrotational. (5marks)

c) A uniform electric field $\boldsymbol{E}$ is at right angles to the magnetic field. Suppose that $\boldsymbol{B}$ points to x-direction and $\boldsymbol{E}$ z-direction. Determine the path taken by a positive charge released from the origin given $y(t) = C_1 \cos \omega t + C_2 \sin \omega t + \left(\frac{E}{B}\right) t + C_3$

$z(t) = C_2 \cos \omega t - C_1 \sin \omega t + C_4$ (10 marks)

QUESTION THREE (20 MARKS)

a) Show that the electric field can be expressed in terms of time varying vector $\boldsymbol{A}$ and a gradient scalar potential $\emptyset$ *i.e* $\boldsymbol{E} = -\nabla\emptyset - \frac{1}{C}\frac{\partial A}{\partial x}$. (5marks)

b) A vector field is given by the expression $\vec{A}(x, y) = a\cos(bx)\,\hat{\imath} + aby\sin(bx)\,\hat{\jmath}$. Show that the field is magnetic. (3marks)

c) Electric charge is uniformly distributed with a linear density $\boldsymbol{\lambda}$ along a semicircle of radius $\boldsymbol{r}$. Determine the Coulomb force on a point charge $\boldsymbol{Q}$ placed at the center of curvature of the semicircle. (4marks)

d) Show that for stationary as well as non-stationary fields, the divergence of the electric field $\nabla . \boldsymbol{E} = 4\pi\rho$ but $\nabla \times \boldsymbol{E}$ has different values. (6marks)

e) A current $\boldsymbol{I}$ is uniformly distributed over a wire of a circular cross section with radius $\boldsymbol{a}$ Find the volume current density $\boldsymbol{J}$. (2marks)

QUESTION FOUR (20MARKS)

a) Use the energy conservation law of electromagnetic fields to show that the flow of energy through a closed surface S surrounding a volume V has an energy flux characterized by a vector $\boldsymbol{\sigma} = \frac{C}{4\pi}(\boldsymbol{E} \times \boldsymbol{H})$. (7marks)

b) Show that the magnitude of torque on an electric dipole placed in an electric field is given by $\boldsymbol{p\vec{E}}\sin\boldsymbol{\theta}$ where the symbols have their usual meaning. (4marks)

c) Determine the meridional component of an electric field for a dipole hence evaluate the field on a broadside-on-position. (5marks)

d) Electric charge is uniformly distributed with a linear density $\boldsymbol{\lambda}$ on an infinitely long line parallel to the y-axis. Determine the electric field strength at a point $\boldsymbol{P}$ a distance $\boldsymbol{b}$ from the center of the segment to the direction of the x-axis. (4marks)

QUESTION FIVE (20 MARKS)

a) Use the poynting theorem to show that the field energy flux flowing through $\mathbf{1}\boldsymbol{cm}^{\mathbf{2}}$ in the direction perpendicular to the field vectors $\boldsymbol{E}$ and $\boldsymbol{H}$ is given by $\boldsymbol{S} = \frac{\mathbf{1}}{\boldsymbol{\mu_o}}(\mathbf{E} \times \mathbf{H})$ (10marks)

b) Find the charge density at $\boldsymbol{x} = \mathbf{2}\boldsymbol{m}$ and $\boldsymbol{x} = \mathbf{5}\boldsymbol{m}$ if the electric field in the region is given by $\vec{\mathbf{E}} = \begin{cases} ax^2 i\,\frac{V}{M}, & 0 \le x \le 3m \\ bi\frac{V}{M}, & x > 3m \end{cases}$ (6marks)

c) Electric charge is uniformly distributed with a density $\boldsymbol{\rho}$ inside a sphere of radius $\boldsymbol{r}$. Determine the electric potential outside the sphere. (4marks)