(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MAP 212/MAP 222/MAT 204**

COURSE TITLE: **REAL ANALYSIS I**

DATE: 25/07/2022 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following terms:

   i. Power set (2marks)
   ii. Cardinality of sets (2marks)
   iii. A discontinuity of a function (2marks)
   iv. A disjoint set (2marks)
   v. Indexed sets (2marks)

b) Let $\mathbb{F}$ be an ordered field and $a \in \mathbb{F}, a \neq 0 \; iff \; a^2 > 0$ (4marks)

c) Show that for $n \geq 1$, $8^n - 3^n$ is divisible by 5 for $n \in \mathbb{N}$. (5marks)

d) Show that an infinite subset of a countable set is countable (4marks)

e) Let $A = \{x: -1 < x < 0\}$ be a subset of the real number field $\mathbb{R}$.

   (i). Categorize the set as either closed or open, explain. (2marks)

   (ii). What is the interior of A? (1mark)

   (iii). Determine the closure, $\overline{A}$, of A. Explain your answer. (2marks)

   (iv). Is A bounded? Explain (2marks)

QUESTION TWO (20 MARKS)

a) State the completeness axiom. (2marks)

b) What do you understand by the term a countable set? (1mark)

c) Prove that the set of integers are countable. (6marks)

d) Define a relation, $x \sim y$, between x and y as $x \equiv y \; mod(n), n \in \mathbb{N}$ meaning that $x - y$ is divisible by n. Show that $x \sim y$ is an equivalence relation. (6marks)

e) Let $A$and B be two finite sets. Show that $(A \cap B)^c = A^c \cup B^c$ (5marks)

QUESTION THREE (20 MARKS)

a) Given that x and y are two positive real numbers, prove that $x < y$ if and only if $x^2 < y^2$ (5marks)

b) Define the following terms

   i. Complete ordered field (2marks)
   ii. Supremum (2marks)
   iii. Complete ordered field (2marks)
   iv. Infimum (2marks)

c) For every real number $x \neq 0$ prove that $x^2 > 0$. (4marks)

d) Given that $X = \{4,5\}$ and $Y = \{-1,0,2\}$, find the Cartesian product of X and Y

   (3marks)

QUESTION FOUR (20 MARKS)

a) Let $n \in \mathbb{N}$. Let $\sim$ be a relation on $\mathbb{N}$ be defined as $x \sim y$ if $x \equiv y \bmod(n)$,that is $x - y$ is divisible by n. Show that $\sim$ is an equivalence relation. (10marks)

b) Let A, B and C be sets. Show that

i. $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ (5marks)

ii. $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$ (5marks)

QUESTION FIVE (20 MARKS)

a) Using the first principle of mathematical induction, show that (5marks)

$$1 + 2 + 3 + 4 + \dots + n = \frac{n\,(n + 1)}{2}, n \in \mathbb{N}.$$

b) Define a closed set, hence, show that arbitrary intersection of closed sets is closed. (6marks)

c) Define a metric space (3marks)

d) Show that $|a| + |b| \geq |a + b|$ (4marks)

e) Define the term Cartesian product of sets X and Y. (2marks)