(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MAP 212/MAP 222/MAT204**

COURSE TITLE: **REAL ANALYSIS I**

DATE: 03/02/2022 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following terms
   i. Disjoint sets (2marks)
   ii. Ordered field (2marks)
   iii. Metric space (5marks)

b) Prove that for some $n \in \mathbb{N}$, $\sum_{k=1}^{n} k^3 = \frac{1}{4}\, n^2(n+1)^2$ (6marks)

c) Prove that every non empty set A of natural numbers has at least element $m \in A$ such that for all $k \in A$, then either $m < k$ or $m = k$ (6marks)

d) Show that $|a| + |b| \geq |a + b|$ (4marks)

e) Show that the power set $P(\mathbb{N})$ of $\mathbb{N}$ is countable (5marks)

QUESTION TWO (20 MARKS)

a) Let $\mathbb{F}$ be a field and $x, y \in \mathbb{F}$. Show that $|x| - |y| \leq |x - y|$. (4marks)

b) Define a function $f: \mathbb{N} \to \mathbb{Z}$ as $f(n) = \begin{cases} \frac{n+1}{2} & where\ n\ is\ odd \\ 1 - \frac{n}{2} & where\ n\ is\ even \end{cases}$. Show that f is a bijection (6marks)

c) Prove that there is no rational number x such that $x^2 = 2$. (6marks)

d) Let $\mathbb{F}$ be an ordered field. Define a metric d on the field as $d(x, y) = \sqrt{|x - y|}$ for $x, y \in \mathbb{F}$. Show d is a metric. (4 marks)

QUESTION THREE (20 MARKS)

a) suppose a relation R in the set of integers is defined as $R = \{(a, b) \mid a - b\ is\ an\ integer\}$. Show that it's an equivalence relation (4marks)

b) Define the following terms
   i. Complete ordered field (2marks)
   ii. Supremum (2marks)
   iii. Infimum (2marks)
   iv. Limit (2marks)

c) Find the infimum, supremum, minimum and maximum of the following sets.
   i. $A = \left(-1, \frac{1}{n}\right), n \in \mathbb{N}$ (4marks)
   ii. $B = \left[\frac{1}{n}, \frac{2+n}{n}\right], n \in \mathbb{N}$ (4marks)

QUESTION FOUR (20 MARKS)

a) State the completeness axiom (2marks)

f) Let A, B and C be sets. Show that

   i. $A\backslash(B \cup C) = (A\backslash B) \cap (A\backslash C)$ (3marks)

   ii. $A\backslash(B \cap C) = (A\backslash B) \cup (A\backslash C)$ (3marks)

b) Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = \frac{3x+2}{7}$ and $g(x) = x^3 - 2x - 3$.
   Find $(g \circ f)(-2)$ (5marks)

c) Differentiate between injective and subjective functions giving examples in each case. (4marks)

d) If $\mathbb{F}$ is an ordered field and $a, b, c \in \mathbb{F}$, show that if $a < b \wedge b < c$ then $a < c$ (3marks)

QUESTION FIVE (20 MARKS)

a) Define the following terms

   i. Bounded set (2marks)

   ii. Equivalence relation (3marks)

a) Prove that a countable union of countable sets is countable (5marks)

b) State the Dedekind axioms (5marks)

c) Show that for $n \geq 1$, $8^n - 3^n$ is divisible by 5 for $n \in \mathbb{N}$. (5marks)