(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAA 411/MAP 412.

COURSE TITLE: COMPLEX ANALYSIS II

DATE: 27/05/2022 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MKS)

(i) Define a power series (2 mks)

(ii) Suppose a power series $\sum a_n Z^n$ converges for $z = z_o \neq 0$. Prove that it converges ;

(a) absolutely for $|z| < |z_o|$ (5 mks)

(b) Uniformly for $|z| \leq |z_1|$ where $|z_1| < |z_o|$ (5 mks)

(iii) Prove that both the power series $\sum_{n=0}^{\infty} a_n Z^n$ and the corresponding series of derivatives $\sum_{n=0}^{\infty} na_n Z^{n-1}$ have the same radius of convergence. (10 mks)

(iv) Find the Laurent series about the indicated singularity for each of the following functions.

(a)$\frac{e^{2z}}{(z-1)^3}$; $z = 1$ (2 mks)

(b)(z-3)sin$\frac{1}{z+2}$; $z = -2$ (2 mks)

(c) $\frac{z-sinz}{z^3}$; $z = 0$ (2 mks)

(d) $\frac{z}{(z+1)(z+2)}$; $z = -2$ (2 mks)

QUESTION TWO (20 MKS)

(a) Define the following terms :

(i) Singular function (2 mks)

(ii) Harmonic function (2 mks)

(b) Find the residue of :

(i) $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ (6 mks)

(ii) $f(z) = e^z csc^2 z$ at all poles in the finite plane (5mks) .

(c) Show that $\int_0^{2\pi} \frac{cos3\theta}{5-4cos\theta} d\theta = \frac{\pi}{R}$. (5 mks)

QUESTION THREE (20 MKS)

(a). Evaluate the following (10 mks)

$\frac{1}{2\pi i} \oint_c \frac{e^{zt}}{z^2(z^2+2z+2)} dz$; around the circle C with equation $|z| = 3$

(b) Define the following terms :

(i) Laurent series (3 mks)

(ii) Principal part (2 mks)

(iii) Analytic part (2 mks)

(iv) Pole of order N (3 mks).

QUESTION FOUR (20 MKS)

(a) Determine by illustration , the region of the W plane into which each of the following is mapped by the transformation $W = Z^2$.

(i) First quadrant of the Z plane .(6 mks)

(ii) Region bounded by $x = 1, y = 1$ and $x + y = 1$. (7 mks)

(b) Find the residue of $f(z) = \frac{e^z}{(z^2+1)z^2}$ (7 mks) .

QUESTION FIVE (20 MKS)

(a) Prove that $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)} = 1$. (15 mks)

(b)Find the residue of $f(z) = \frac{4-3z}{z^2-z}$ (5 mks)