(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SUPPLEMENTARY/SPECIAL EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATON AND BACHELOR OF SCIENCE

COURSE CODE: MAA 225

COURSE TITLE: COMPLEX ANALYSIS I

DATE: 18/07/2022 **TIME:** 11 AM -1 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define and give an example of following types of singularities

(i) Poles (2 mks)

(ii) Removable (2 mks)

(iii) Essential (2 mks

b) Using De Moivre's theorem show that

$sin4x = 4(sinxcos^3x - sin^3xcos\ x)$ (4 mks)

c) (i) Find all the values of x, for which $x^4 + 81 = 0$ (5 mks)

(ii) Locate these values in (i) above in a complex plane (3 mks)

d) Find the residuals of $f(z) = \frac{2z+1}{z^2-z-2}$ at all its poles and hence evaluate

$\oint_C \quad f(z)dz$ (6 mks)

e) Use the Milne-Thomson method to find a function $U(x, y)$ such that

$f(z) = U(x, y) + iV(x, y)$, given that $U(x, y) = 2x + \frac{3x}{x^2+y^2}$ (6 mks)

QUESTION TWO (20 MARKS)

a) Given that $w = f(z) = 2z(1 + z)$ find the values of w corresponding to

$z =- 3 - 4i$ (4 mks)

b) (i) State the Taylors series of expansion (2 mks)

(ii) Find the first four terms of the Taylor series expansion for the function

$f(z) = \frac{z+2}{(z-1)(z-3)}$ about the point $z = 2$ (6 mks)

c) Evaluate $\oint_C \quad \frac{z+2}{z^3-9z}dz$ where C is the circle

(i) $|z + 3| = \frac{5}{2}$ (4 mks)

(ii) $|z - 3| = \frac{5}{2}$ (4 mks)

QUESTION THREE (20 MARKS)

a) Prove that $cos\ cos\ (z_1 + z_2) = cosz_1cosz_2 - sinz_1sinz_2$ (5 mks)

b) Show that the function $V(x, y) = e^x(xcosy - ysiny)$ is harmonic (5 mks)

c) Evaluate $\int_{-2+2i}^{4i} \quad (x^2 - 2y)dx - (4x + y)dy$ along

i) The parabola $x = 3t, y = 2t^2 - 2$ (5 mks)

ii) The straight line from $(0,2)$ to $(3,5)$ (5 mks)

QUESTION FOUR (20 MARKS)

a) Convert into polar form $z = 3i - 4$ (3 mks)

b) State the Laurent series (2 mks)

c) Find the Laurent series expansion in the region $1 < |z| < 3$ for

$f(z) = \frac{1}{(z+1)(z+3)}$ (7 mks)

d) Find the analytic function $w = f(z)$ if its real part is $U(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$ and if $f(-1) = i - 2$ (8 mks)

QUESTION FIVE (20 MARKS)

a) Evaluate $\frac{(2z-3)(3z+i)}{(1-iz)^2}$ (3 mks)

b) Define the following terms

(i) Simply connected region (2 mks)

(i) Multiply connected region (2 mks)

c) Verify Green's theorem in the plane for $\oint_C \quad (2y^2 - xy)dx + (3x^2 - x^3y)dy$ Where C whose vertices are $(0,0)$, $B(4,0)$, $C(4,4)$ and $D(0,4)$ (6 mks)

d) Use Cauchy's integral formula to evaluate $\int_C \quad \frac{2z^2+z}{z^2-1}dz$ where C is the circle $|z - 1| = 1$ (7 mks)