KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR

MAIN EXAMINATION

SECOND YEAR FIRST SEMESTER EXAMINATIONS
FOR THE DEGREE
OF
BACHELOR OF SCIENCE.

FINAL EXAMINATIONS

COURSE CODE: **SPH 211**

COURSE TITLE: **WAVES & OSCILLATIONS**

DATE: 14/12/2022 **TIME:** 2:00-4:00PM

INSTRUCTIONS TO CANDIDATES

- Question ONE is compulsory and carries 30 marks
- Attempt any two of the remaining questions. Each carries 20 marks.

QUESTION ONE (30 MARKS)

a. Name with examples the classification of waves. (2.mks)

b. Given that the motion of a point x at time t is same as the motion of the point x = 0 at an earlier time $\left(t - \frac{x}{V}\right)$ for a sinusoidal wave $y(x,t) = Asin\omega t$ and propagation constant $k = \frac{2\pi}{\aleph}$ show that $\omega = vk$. (2.mks)

c. For a particle with a displacement of $y = Asin(\omega t - kx)$, determine equation for;
(i). velocity of the particle
(ii). acceleration of the particle. (2.marks)

Hence show that the wave equation with V as the wave speed is given as

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\delta^2 y}{\delta x^2}$$

(3.marks)

d. (i) Show that the propagation velocity is given as

$$V = \sqrt{\frac{T}{\mu}}$$

(3.marks)

e. A cloth line has a linear mass density of 25Kg/m^3 and is stretched with a tension of 2500N. One end is given a sinusoidal motion with frequency of 500Hz and amplitude 1m. At a time t=0, the end has zero displacement and moving in +y direction.

i. Find the speed, amplitude, angular frequency, period, wavelength and wave number. (6.marks)
ii. Write a wave function describing the wave (1.marks)
iii. Find the position of the point x=25m and time t=15s (2marks)
iv. Find the transverse velocity at x=2.5 at t=15s (2.marks)
v. Find the slope of the string at x=25at t=10s (2.marks)

f. (i) Show that $V = \sqrt{\frac{B}{\rho}}$ where B is the bulk modulus

(ii) Determine the speed of sound waves in water and find a wavelength of a wave having a frequency 262Hz (k=45.8 x 10^{-11}) Pa, ρ=1000Kg/m^3. (2.marks)

(iii) What is the speed of longitudinal wave in a steel rod, given Y= 2x10^{11}, p = 7.8 x10^{3}Kg/m^3 (2.marks)

QUESTION TWO (20MARKS)

a. Define simple harmonic motion (SHM) (1mark)

b. State two basic properties that course oscillation of a physical system. (2smarks)

c. Name three main characteristics of a simple harmonic motion (3marks)

d. In a motion in which the restoring force is proportional to the displacement from mean position that opposes its increase, show that, the general equation of simple harmonic motion is given as

$\ddot{Y} = -\omega^2 Y$ and $\omega^2 = \frac{K}{M}$ (4marks)

e. If $\varphi = \frac{\pi}{2}$ and $x = A\cos\left(\omega t + \frac{\pi}{2}\right)$, using the identity $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$, show that $X = -\sin\omega t$ (6marks)

f. Name three examples of SHM of systems with one degree of freedom, hence show that for a simple pendulum $\omega = \sqrt{\frac{g}{l}}$ (4marks)

QUESTION THREE (20 MARKS)

a. Consider two superposed vibrations of equal frequency as

$$X_1 = a_1\cos(\omega t + \alpha_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

$$X_2 = a_2\cos(\omega t + \alpha_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

And $X = X_1 + X_2$

Show that

i. Phase angle of a wave is given as

$$\tan\theta = \frac{a_1\sin\alpha_1 + a_2\sin\alpha_2}{a_1\cos\alpha_1 + a_2\cos\alpha_2}$$

(10marks)

ii. The resultant amplitude is given as (6marks)

$R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos(\alpha_1 - \alpha_2)}$

iii. And equation of displacement is given as (4marks)

$X = R\cos(\omega t + \theta)$

QUESTION FOUR. (20 Marks)

a) A block of mass m = 100 g attached to a horizontal spring with k = 0.4 N/m is in simple harmonic motion with a displacement given by $x = -0.2\cos(\omega t)$. Calculate the period, T, and the velocity of the block at time $t = 3T/8$. [3 marks]

b) Write down the differential equation that characterizes any simple harmonic oscillation, and its general solution. List the three conditions that must be satisfied for simple harmonic motion to occur in a mechanical system. [3 marks]

c) A particle of mass 100 g executes simple harmonic motion about x = 0 with angular frequency $\omega = 10\ s^{-1}$. Its total mechanical energy is $E_{tot} = 0.45$ J. Find the displacement of the particle when its speed is 2 m/s. [3 marks]

d) A damped oscillator is driven by a force $F(t) = F_0\cos(\omega_e t)$. Explain briefly what is meant by the *transient* and the *steady-state* solutions for the displacement x(t), and write down the

general form of the steady-state solution. [3 marks]

e) State the principle of linear superposition and give two examples of physical phenomena that rely on it. [3 marks]

f) Write down the general form for a harmonic wave travelling along the x axis, in terms of k, ω and ϕ_0. Determine the wavelength, frequency, phase constant, and phase speed of the wave

$$y(x,t) = 0.5\sin\left(0.2\pi x + 3\pi t + \frac{\pi}{6}\right),$$

in which the units of x and y are metres and t is in seconds. [2 marks]

g) A violin string must be tuned to vibrate at a frequency of 660 Hz in its fundamental mode. The vibrating part of the string is 33 cm long, and the linear density is 3 g/m. What is the tension when the string is in tune? [3 marks]

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QUESTION FIVE. (20 Marks)

A particle of mass m is in simple harmonic motion about an equilibrium position x = 0, with an angular frequency ω.

(a) Write down the differential equation of motion for the particle. Give the general solution for $x(t)$, and its first two derivatives, $\dot{x}(t)$ and $\ddot{x}(t)$. [4 marks]

(b) If the potential energy is defined to be U = 0 at equilibrium, show that $U = \frac{1}{2}m\omega^2 x^2$ in general. (Hint: use the work-energy theorem.) [4 marks]

(c) Use the results from (a) and (b) and the general definition of kinetic energy K, to find K and U as functions of time. Use these to find an expression for the total Mechanical energy, Etot = K + U. [4 marks]

(d) Obtain a formula for the kinetic energy in terms of m, ω, x and the amplitude of oscillation. Find the displacement x at which K = U. [4 marks]

(e) Sketch the dependence of K, U, and Etot on displacement x. Label the minimum and maximum values of all quantities. [4 marks]