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*(Knowledge for Development)*

**KIBABII UNIVERSITY**

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**UNIVERSITY EXAMINATIONS**

**2021/2022 ACADEMIC YEAR**

**FIRST YEAR FIRST SEMESTER**

**MAIN EXAMINATION**

**FOR THE DEGREE OF BACHELOR OF SCIENCE**

**COURSE CODE:** STA 112/STA 142

**COURSE TITLE:** INTRODUCTION TO PROBABILITY

**DATE:** 04/02/2022

**TIME:** 2:00 PM - 4:00 PM

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**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

**QUESTION ONE (30 MARKS)**

1. (a) Define the following terms: Sample outcome, finite set, exhaustive events (3 mks)
- (b) For two events A and B, suppose that  $P(A) = 0.3$ ,  $P(B) = 0.5$ , and  $P(A \cup B) = 0.6$ . Calculate  $P(A \cap B)$ . (3 mks)
- (c) Let A be the set of x's for which  $x^2 + 2x = 8$ ; let B be the set for which  $x^2 + x = 6$ . Find  $A \cap B$  and  $A \cup B$  (4 mks)
- (d) i. Explain the following: discrete random variable and continuous random variable (2 mks)
- ii. A random variable X has the probability distribution below

X	0	1	2	3	4	5	6	7	8
P(X=x)	a	2a	3a	4a	5a	6a	7a	8a	9a

- A. Determine the value of a (2 mks)
- B. Find  $P(X < 3)$ ,  $P(0 < X < 5)$  (4 mks)
- (e) A committee of 4 people need to be selected from 5 women and 7 men. How many ways can the committee be selected if at least 3 women must be included. (3 mks)
- (f) A fair die is thrown twice. A is the event "sum of the throws equals 4," B is "at least one of the throws is a 3." Calculate  $P(A|B)$ ; Are A and B independent? (3 mks)
- (g) If two fair dice are rolled once, what is the probability that the total number of spots shown is equal to 5? (3 mks)
- (h) At a party with 10 guests, every guest shakes hands with each other. How many handshakes can be counted in total? (3 mks)

## QUESTION TWO (20 MARKS)

2. (a) Let  $X$  be random variable with pdf

$$f(x) = \begin{cases} \frac{x}{5}, & x = 1, 2, 3, 4 \\ 0, & \text{elsewhere} \end{cases}$$

Compute;

- i.  $E(X)$ , (2 mks)
  - ii.  $E(3X)$  (3 mks)
  - iii.  $Var(X)$  (3 mks)
- (b) Kibabii university library subscribes to three news magazines A, B, and C according to the following proportions: A : 20%, B : 15%, C : 10%, both A and B : 5%, both A and C : 4%, both B and C : 3%, all three A, B, and C : 2%. If a student is chosen at random, what is the probability he/she subscribes to none of the news magazines? (6 mks)
- (c) Let  $A, B \in S$ , show that
- i.  $P(A) = P(A) - P(A \cap B)$  (3 mks)
  - ii.  $P(A') = 1 - P(A)$  (3 mks)

## QUESTION THREE (20 MARKS)

3. (a) A university library has five copies of a textbook to be used in a certain class. Of these copies, numbers 1 through 3 are of the 1st edition, and numbers 4 and 5 are of the 2nd edition. Two of these copies are chosen at random to be placed on a 2-hour reserve.
- i. Write out an appropriate sample space  $S$ . (2 mks)
  - ii. Consider the events A, B, C, and D, defined as follows, and express them in terms of sample points.  
A = both books are of the 1st edition, (2 mks)  
B = both books are of the 2nd edition, (2 mks)  
C = one book of each edition, (2 mks)  
D = no book is of the 2nd edition. (2 mks)
- (b) Let  $X$  have the pdf

$$f(x) = \begin{cases} \frac{1}{2}(x+1), & -1 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find

- i.  $Var(X)$  (5 mks)
- ii.  $Var(5X)$  (5 mks)



#### QUESTION FOUR (20 MARKS)

4. (a) Consider tossing two fair dice. Let  $X$  denote the sum of the upturned values of the two dice and  $Y$  their absolute difference. Calculate the expected value of  $X$  and  $Y$ . (8 mks)
- (b) Let  $X$  (in tonnes) be a random variable representing the quantity of sugar sold in a day at a certain factory with a distribution function as shown;

$$f(x) = \begin{cases} cx, & 0 \leq x \leq 3 \\ c(10 - x), & 3 < x \leq 5 \\ 0, & \text{elsewhere} \end{cases}$$

- i. Find  $c$  such that  $f(x)$  is a pdf (4 mks)
- ii. Find  $P(X \leq 3)$  (2 mks)
- iii. Find  $P(X > 3)$  (2 mks)
- iv. Find  $P(2.5 \leq X \leq 5)$  (4 mks)

#### QUESTION FIVE (20 MARKS)

5. (a) Two boxes each contain three cards. The first box contains cards labeled 1, 3 and 5. The second box contains cards labeled 1, 3, and 5. In a game, a player draws one card at random from each box and his score,  $X$ , is the sum of the numbers on the two cards.
- i. Obtain the six possible values of  $X$  and find their corresponding probabilities (2 mks)
- ii. Calculate the standard deviation of  $X$ . (6 mks)
- (b) A driving licence examination consists of two parts which are based on a theoretical and a practical examination. Suppose 25% of people fail the practical examination, 15 % of people fail the theoretical examination, and 10 % of people fail both the examinations. If a person is randomly chosen, then what is the probability that this person
- i. fails at least one of the examinations? (3 mks)
- ii. only fails the practical examination, but not the theoretical examination? (3 mks)
- iii. successfully passes both the tests? (3 mks)
- iv. fails any of the two examinations? (3 mks)