(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

COURSE CODE: MAP 211/MAT 201

COURSE TITLE: LINEAR ALGEBRA I

DATE: 25/01/2022 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE

a) Show that:

I. The set of all 2x3 matrices with the operations of matrix addition and scalar multiplication is a vector space. (4 Marks)

II. The subset of IR^2 consisting of all points on the unit circle $x^2+y^2=1$ is not a subspace. (4 Marks)

b) If **u** and **v** are vectors, show that **u**+**v** = **v**+**u** (4 Marks)

c) solve the following system of equations using Gauss-Jordan elimination method

$$x+y+z = 5$$
$$2x+3y+5z = 8$$
$$4x+5z = 2$$

(6 marks)

d) Express v = (3,7,-4) as a linear combination of $u_1 = (1,2,3)$, $u_2 = (2,3,7)$, $u_3 = (3,5,6)$ (4 marks)

e) $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, and $S = \{e_1, e_2, e_3\}$. Are e_1, e_2 and e_3 linearly independent? (3 marks)

F) Let $L : R^3 \to R^3$,

$$L(x) = Ax = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Is L onto? (5 marks)

QUESTION TWO

a) Prove that If $S = \{V_1, V_2, \ldots V_n\}$ is a basis for vector space V then every set with more than n vectors of V is linearly dependent. (10 Marks)

b) Let S = { (1,2), (2,4), (2,1),(3,3) (4,5) } show that IR^2 = span S (10 Marks)

QUESTION THREE

a) Let **u** = (1, 2, 3), **v** = (2,-3, 1) and **w** = (3, 2,-1)

i) Find the components of the vector **u**-3**u**+8**w** (2 marks)

ii) Find the scalars c_1, c_2, c_3 such that c_1**u**+c_2**v**+c_3**w** = (6, 14,-2) (6 marks)

b) How is linear independence and linear dependence tested? (5 marks)

b) Determine whether the following set of vectors is linearly independent or not. v_1 = (1, 2, 3), v_2 = (0, 1, 2), v_3 = (-2, 0, 1) (7 marks)

QUESTION FOUR

a) Is the vector $v = \begin{bmatrix} 3 \\ -4 \\ -6 \end{bmatrix}$ a linear combination of the vectors

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}.$$

(10 marks)

b) Let

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix},\ v_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix},\ v_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},\ \text{and}\ S = \{v_1, v_2, v_3\}.$$

Does $span(S) = R^3$? (10 marks)

QUESTION FIVE

a) Show that P, the vector space of all polynomials cannot be spanned by a finite set of polynomials (8 marks)

b) Solve (6 marks)

$4x_1+8x_2-12x_2=44$

$3x_1+6x_2-8x_3=32$

$-2x_1-x_2=-7$

c) Use Gauss-Jordan elimination method to solve (6 marks)

$x_1+x_2+x_3=2$

$2x_1+3x_2+x_3=3$

$x_1-x_2-2x_3=-6$