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(Knowledge for Development)

# KIBABII UNIVERSITY

## UNIVERSITY EXAMINATIONS

## 2022/2023 ACADEMIC YEAR

## SECOND YEAR FIRST SEMESTER

## MAIN EXAMINATION

## FOR THE DEGREE BACHELOR OF SCIENCE

**COURSE CODE:** **MAA 211/ MAT 203**

**COURSE TITLE:** **VECTOR ANALYSIS**

**DATE:** 19/12/2022 **TIME:** 9:00 AM - 11:00 AM

## INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

**QUESTION ONE COMPULSORY (30 MARKS)**

a) Given $\vec{A} = 4\boldsymbol{i} - 2\boldsymbol{j} + 2\,\boldsymbol{k}$ and $\vec{B} = 2\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k}$, find $\overrightarrow{A} \cdot (\overrightarrow{\vec{A} \times \vec{B}})$ (3 marks)

b) Prove the associative law for vector addition. (4 marks)

c) If $\vec{U} = 4\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$ and $\vec{V} = \boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k}$, find

   i. $\vec{U} \times \vec{V}$

   ii. $(\vec{V} - \vec{U}) \times (\vec{V} + \vec{U})$ (6 marks)

d) Find the magnitude of moment of force $\vec{F} = 3\boldsymbol{i} + \boldsymbol{k}$ about the point $(1, -3, 3)$ whose line of action passes through the origin. Sketch it. (5 marks)

e) If $\vec{A} = A_1\boldsymbol{i} + A_2\boldsymbol{j} + A_3\,\boldsymbol{k}$ and $\vec{B} = B_1\boldsymbol{i} + B_2\boldsymbol{j} + B_3\,\boldsymbol{k}$, prove that

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}.$$

(4 marks)

f) If $\vec{R} = (5x^2 + y^2)\boldsymbol{i} + 3xy\,\boldsymbol{j} + (x^2y - 2z^2)\,\boldsymbol{k}$, find

   i. Divergence of $\vec{R}$

   ii. Curl of $\vec{R}$ (4 marks)

g) Given $\vec{V} = (3x^3y - x^3)\boldsymbol{i} + (e^{xy} - y\cos 3x)\boldsymbol{j} + \sin y\,\boldsymbol{k}$, determine

   i. $\frac{\partial \vec{V}}{\partial x}$ at $(0,1)$

   ii. $\frac{\partial \vec{V}}{\partial y}$ at $(1,0)$ (4 marks)

**QUESTION TWO (20 MARKS)**

a) Determine a unit vector perpendicular to the plane that contains $\vec{A} = 2\boldsymbol{i} - 6\boldsymbol{j} - 3\,\boldsymbol{k}$ and $\vec{B} = 4\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ (3 marks)

b) Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{A} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $\vec{B} = 3\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}$ (3 marks)

c) Show that $div$ curl $\vec{A} = 0$ where $\vec{A}$ is a vector field which has continuous second partial derivatives. (4 marks)

d) Find the volume of the parallelopiped whose adjacent edges are $\vec{a} = \boldsymbol{i} + 2\boldsymbol{j} + 5\,\boldsymbol{k}$, $\vec{b} = 4\boldsymbol{i} + 6\boldsymbol{j} + 2\boldsymbol{k}$ and $\vec{c} = 2\boldsymbol{i} + 3\boldsymbol{j} - 6\,\boldsymbol{k}$. (4 marks)

e) A particle moves along the curve $x = t^2 - 4t, y = 2t^2, z = 3t - 5$, where $t$ is time. Find the components of its velocity and acceleration at time $t = 1$ in the direction of $\boldsymbol{i} - 3\boldsymbol{j} + 2\,\boldsymbol{k}$ (6 marks)

## QUESTION THREE (20 MARKS)

a) Define the terms

  i. divergence of a vector field

  ii. grad of a scalar field (4 marks)

b) Find the work done in moving an object along a straight line from $(2,-1,-4)$ to $(3,2,-1)$ in a force field given by $\vec{F} = 4\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$ (3 marks)

c) Find the value of m for which $\vec{a} = m\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ and $\vec{b} = 2m\boldsymbol{i} + m\boldsymbol{j} - 4\,\boldsymbol{k}$ are perpendicular. (2 marks)

d) Find the directional derivative of $\emptyset = x^2yz + 4xz$ at $(1,-2,-1)$ in the direction of $2\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$. (4 marks)

e) Given $\vec{F} = x^2y\boldsymbol{i} - 2xz\boldsymbol{j} + 2yz\boldsymbol{k}$, find curl $\vec{F}$ (3 marks)

f) If $\emptyset = \frac{1}{\sqrt{x^2+y^2+z^2}}$ find gradient of $\emptyset$. (4 marks)

## QUESTION FOUR (20 MARKS)

a) Differentiate between irrational vector and solenoidal vector. (2 marks)

b) Determine the constant $\beta$ so that $\vec{V} = (x+3y)\boldsymbol{i} + (y-2z)\boldsymbol{j} + (x-\beta z)\boldsymbol{k}$ is a solenoidal vector field. (2 marks)

c) If $\vec{A}$ and $\vec{B}$ are differentiable vector functions of a scalar $t$, show that

$$\frac{d(\vec{A}\cdot\vec{B})}{dt} = \vec{A}\cdot\frac{d(\vec{B})}{dt} + \frac{d(\vec{A})}{dt}\cdot\vec{B}$$ (3 marks)

d) Given the force field, $\vec{F} = (x+2y+4z)\boldsymbol{i} + (2x-3y-z)\boldsymbol{j} + (4x-y+2z)\boldsymbol{k}$

  i. Show that $\vec{F}$ is a conservative force field (3 marks)

  ii. Find the scalar potential (5 marks)

e) Find the area of a triangle with vertices at $A(4,2,-1)$, $B(2,3,5)$ and $C(3,6,4)$. (5 marks)

**QUESTION FIVE (20 MARKS)**

a) State without proving the Green's Theorem in the plane. (3 marks)

b) If $\vec{R}(t) = (t - t^2)\boldsymbol{i} + 2t^3\boldsymbol{j} - 3\boldsymbol{k}$. Find, $\int_1^2 \vec{R}(t)\, dt$ (4 marks)

c) Verify the Green's Theorem in the plane for $\oint_C (xy + y^2)dx + x^2\, dy$ where $C$ is the closed curve of the region bounded by $y = x$ and $y = x^2$ (7 marks)

d) Given $\vec{A} = 2z\boldsymbol{i} - x\boldsymbol{j} + y\,\boldsymbol{k}$, evaluate $\iiint_v \vec{A}\, dV$ where $V$ denote the region bounded by the surfaces $x = 0, y = 0, x = 2, y = 6, z = x^2, z = 4$ (6 marks)