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*(Knowledge for Development)*

**KIBABII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**  
**2021/2022 ACADEMIC YEAR**  
**FIRST YEAR FIRST SEMESTER**  
**MAIN EXAMINATION**

**FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE**

**COURSE CODE: MAP 112**

**COURSE TITLE: BASIC MATHEMATICS AND ANALYTIC  
GEOMETRY**

**DATE: 26/01/2022**

**TIME: 9:00 AM - 11:00 AM**

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**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

### QUESTION ONE (30 MARKS)

- a) Give the parametric forms of equations of
- a parabola which opens downwards with centre at the origin
  - a hyperbola with a horizontal orientation with centre at  $(h, k)$
  - an ellipse with with centre at  $(h, k)$
- (3 marks)
- b) Find the points on the line  $x + 3y = 4$  which lie 3 units from the line  $4x + 3y = 10$ . (5 marks)
- c) Find the point and angle of intersection of the lines  $5x - 6y - 1 = 0$  and  $3x + 2y + 5 = 0$  (4 marks)
- d) Convert  $r = \frac{3}{1-2 \cos \theta}$  to Cartesian form and identify the centre of the graph. (5 marks)
- e) Find the equation of a parabola, directrix and its sketch having its vertex at  $(-1, 2)$  and focus at  $(0, 2)$ . (4 marks)
- f) Find the directrix, eccentricity and sketch the graph of  $r = \frac{250}{100 + 100 \sin \theta}$  (4 marks)
- g) Convert the following equation to Cartesian form, giving its vertex and sketch. (5 marks)
- $$x = t^2 + t, y = 2t - 1 \quad -3 \leq t \leq 3$$

### QUESTION TWO (20 MARKS)

- a) Show that  $\cosh^2(x) - \sinh^2(x) = 1$  (2 marks)
- b) Find the zeros of  $f(x) = x^3 - 6x^2 + 11x - 6$  (4 marks)
- c) Show that two circles  $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$  and  $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$  are said to be orthogonal if
- $$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$
- (7 marks)
- d) Write an equation for a parabola with focus at  $(2, 1)$  and directrix  $x = 6$ . Sketch its graph. (5 marks)

e) Using trigonometric substitution convert the following equation into parametric form.

$$\frac{x^2}{49} + \frac{y^2}{25} = 1 \quad (2 \text{ marks})$$

### QUESTION THREE (20 MARKS)

a) Define the following terms

- i. Eccentricity
  - ii. Hyperbola
- (4 marks)

b) Find the distance between the lines  $3x + 4y = 9$  and  $6x + 8y = 15$  (3 marks)

c) The moon orbits the Earth in an elliptical path with the center of the Earth at one of the focus. The major and minor axes of the orbit have the lengths of 768,800 km and 767,640 km respectively. Find the greatest and least distances from the Earth's center to the moon's center. Give the respective equation and sketch of the motion. (6 marks)

d) Convert the polar equation given below to Cartesian equation.

- i.  $r = \sin(2\theta)$  (3 marks)
- ii.  $\theta = \beta$  (2 marks)
- iii.  $(-1, \frac{2\pi}{3})$  (2 marks)

### QUESTION FOUR (20 MARKS)

a) Sketch and derive the equation  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  (8 marks)

b) Analyze and sketch the graph of  $(x + 2)^2 = -12(y + 1)$  (5 marks)

c) Convert the following set of parametric equation to Cartesian equation.

$$x = 6 \cos t, y = 2 \sin t, 0 \leq \theta \leq 2\pi. \quad (3 \text{ marks})$$

d) Convert the following equation to polar  $(x - a)^2 + (y - b)^2 = a^2 + b^2$  (4 marks)

**QUESTION FIVE (20 MARKS)**

a) Analyze the following equations

i.  $12x^2 + 20y^2 - 12x + 40y - 37 = 0$  (6 marks)

ii.  $9y^2 - x^2 + 2x + 54y + 62 = 0$  (6 marks)

b) Find the vertex, axis of symmetry, focus, directrix and sketch the graph of the parabola  $y^2 + x + y = 0$  (4 marks)

c) Verify whether the circles  $x^2 + y^2 - 4x + 6y + 6 = 0$  and  $x^2 + y^2 - 4x - 6y + 4 = 0$  touch each other. (4 marks)