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*(Knowledge for Development)*

**KIBABII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**  
**2021 /2022 ACADEMIC YEAR**  
**FIRST YEAR FIRST SEMESTER**  
**MAIN EXAMINATION**

**FOR THE DEGREE OF BACHELOR OF SCIENCE**

**COURSE CODE:** MAA 111/MAT 121

**COURSE TITLE:** DIFFERENTIAL CALCULUS/CALCULUS 1

**DATE:** 01/02/2022

**TIME:** 2:00 PM - 4:00 PM

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**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

**TIME:** 2 Hours

### QUESTION ONE (30 MARKS)

- (a) Given the functions  $f(x) = x^2 + 2x$  and  $g(x) = x - 2$ , find
- (i)  $fg(x)$  (2 Marks)
  - (ii)  $fg(-2)$  (1 Mark)
- (b) Given the function  $\frac{\sqrt{x^3-27}}{x(x-1)}$  find its
- (i) Domain (3 Marks)
  - (ii) Range (2 Marks)
- (c) Evaluate  $\lim_{x \rightarrow 1} \frac{x^3-1}{x-1}$  (3 Marks)
- (d) Given  $y = 4e^{2x} \sin 4x$ , find  $\frac{dy}{dx}$  (5 Marks)
- (e) Find  $y'$  if  $3x^2 - 4xy^2 = \sin y$  (5 Marks)
- (f) Find the stationary points to the curve  $y = 8 + 3x - 3x^2 + x^3$  and identify their nature (6 Marks)
- (g) Given  $x = \sin t$  and  $y = t^2 - t$ , find  $\frac{dy}{dx}$  (3 Marks)

### QUESTION TWO (20 MARKS)

- (a) Given the function  $f(x) = 4x + 2x^2$ , find
- (i) Find  $f'(x)$  from the first principles (6 Marks)
  - (ii) Hence evaluate  $f'(3)$  (3 Marks)
- (b) Given the function  $y = 2x^2 - 5x$ , find at  $(-1, 1)$ , the equation of the
- (i) Tangent to the function (4 Marks)
  - (ii) Normal to the function (3 Marks)
- (c) Find the slope of the curve  $y = 0.2x^2 - x^{-3} + 0.1$  at the point  $x = -4$  (4 Marks)

### QUESTION THREE (20 MARKS)

- (a) If  $\cos(x + y) = y^2 \sin x$ , find  $\frac{dy}{dx}$  (7 Marks)
- (b) Determine the gradient of the normal to the graph of  $xy^2 + 4xy + 7x = 4$  at the point  $(2, -2)$  (7 Marks)
- (c) Find  $\frac{d^3y}{dt^3}$  given that  $y = 3t^2 + 10t^{\frac{2}{5}} + e^{2t}$  (6 Marks)

**QUESTION FOUR (20 MARKS)**

(a) Evaluate  $\lim_{x \rightarrow 0} \frac{e^{-4x} + 1}{1 + \cos x}$  (7 Marks)

(b) Find  $\lim_{x \rightarrow \infty} \frac{3x^2 + 4x - 5}{\sqrt{x^6 - 1}}$  (6 Marks)

(c) Evaluate  $\lim_{t \rightarrow -3} \frac{t^2 - t - 12}{t + 3}$  (6 Marks)

**QUESTION FIVE (20 MARKS)**

(a) Given  $y = \frac{3\cos 2x}{2x^2}$ , find  $\frac{dy}{dx}$  (6 Marks)

(b) The position of a moving particle is given by the equation  $s = 3t^3 - t^2 + 9t$  where  $t$  is in seconds and  $s$  is in metres. Find the velocity

(i) After 3 seconds

(ii) When the particle is at rest

(iii) Find the acceleration after 5 seconds (7 Marks)

(c) A rectangular plot is to be enclosed by a fencing wire 500m long. What is the maximum possible area that can be enclosed? (7 Marks)