(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAP 313

COURSE TITLE: GROUP THEORY

DATE: 15/07/21 **TIME:** 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

END OF SEMESTER EXAMINATIONS

YEAR TWO SEMESTER ONE EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

COURSE CODE : MAP 313

COURSE TITLE : GROUP THEORY I

DATE **TIME:**

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTIONS ONE AND ANY OTHER TWO

QUESTION ONE (30 MARKS)

a. Define the following
   i. Abelian group (1mark)
   ii. Subgroup (3marks)
   iii. Coset (2marks)
   iv. Normal subgroup (2mark)

b. Let G be a group and $a, b \in G$. Show that the equation $ax = b$ has a unique solution (5 marks)

c. Let G be a group. Show that $x * z = y * z \Rightarrow x = y$ for $x, y \in G$ (4 marks)

d. Let K be the subgroup of s_3 defined by the permutations {(1), (12)} determine the right cosets of K in S_3 (7marks)

e. Let H be a subgroup of a group G. Show that the left cosets of H in G partition G. (6marks)

QUESTION TWO (20 MARKS)

a. Define the following
   i. Center of a group (2marks)
   ii. Homomorphism (2marks)
   iii. Automorphism (2marks)
   iv. Isomorphic groups (2marks)

b. Let $\varphi: G \rightarrow H$ be a homomorphism, and let e,e$^{/}$ denote the identity elements of G and H respectively. Show that
   i. $\varphi(e) = e^{/}$ (1mark)
   ii. $\varphi(a^{-1}) = \varphi(a)^{-1}$ (1mark)

iii. $\varphi(a^n) = \varphi(a)^n$ (2marks)

c. Show that φ is a monomorphism if and only if $\ker \varphi = \{e\}$ (6marks)

d. Given $g \in G$ define the map $\varphi: G \to G$ by $\varphi(a) = gag^{-1}$ Show that $\varphi \in AutG$. (4marks)

e. Give two examples of homomorphisms (4marks)

QUESTION THREE (20 MARKS)

a. Define the following
   i. Permutation (1mark)
   ii. Symmetric group (2marks)
   iii. Transposition (2marks)
   iv. Even and odd transposition (2 marks)
   v. Alternating group (2marks)

b. Write the following cycle notations of S_4 in permutations
   i. (12)(34) (1mark)
   ii. (23) (1mark)
   iii. (13)(24) (1mark)
   iv. (132) (1mark)

c. Compose the following cycle notation (1234)∗(13)(24) (4marks)

d. Show that every permutation can be expressed as a product of transpositions (2 marks)

e. Represent the permutation $(13584)(2967) \in S_9$ as a product of transpositions (1mark)

QUESTION FOUR (20 MARKS)

a. Define the following
   i. Normal subgroup (2marks)
   ii. Simple group (1mark)
   iii. Composition series (3marks)
   iv. Cyclic group (3 marks)

b. Show that every cyclic group is abelian (4 marks)

c. Show that every subgroup of a cyclic group is cyclic (7marks)

QUESTION FIVE (20 MARKS)

a. Define the following
   i. Group action (3marks)
   ii. Orbit (2marks)
   iii. Stabilizer (2marks)

b. Show that stab(x) is a subgroup of G for each $x \in X$ (5marks)

c. Show that the orbits of an action partition X (4marks)

d. Show that if $\lceil G \rceil = n$, then there is an embedding $G \to S_n$ (4 marks)