



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR
FIRST YEAR SECOND SEMESTER
SUPPLEMENTARY EXAMINATION
FOR THE DEGREE OF BACHELOR OF INFORMATION
TECHNOLOGY

COURSE CODE: MAT 121

COURSE TITLE: LINEAR ALGEBRA I

DATE: 25/07/2022

TIME: 8:00 AM - 10:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE COMPULSORY (30 MARKS)

- a) Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 1 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ 1 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ 0 & 3 \end{bmatrix}$
- Find A^{-1} . (3marks)
 - Find a matrix X such that $AX + B = C$. (4marks)
 - Is it possible to find a matrix Y such that $YA + B = C$? Explain. (4marks)
- b) Let $A = \begin{bmatrix} 7 & 3 \\ -2 & -5 \end{bmatrix}$ and $X = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$. Compute, if possible, $X^T A^T$, $A^T X^T$, $X^T X$, and XX^T

(10marks)

- c) Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Find C if $AC = B^T$. (5marks)
- d) Find a vector X , of length 6, in the opposite direction of $Y = (1, 2, -2)$. (4marks)

QUESTION TWO (20 MARKS)

- a) Solve the system (6marks)

$$\begin{aligned}x + y + z &= 0 \\x + 2y + 3z &= 0 \\x + 3y + 4z &= 0 \\x + 4y + 5z &= 0.\end{aligned}$$

- b) Show that if $A^{-1} = A^T$, then $|A| = 1$ or $|A| = -1$ (3marks)
- c) If A and B are 2×2 matrices with $\det(A) = 2$ and $\det(B) = 5$, compute $|3A^2(AB^{-1})^T|$ (5marks)
- d) Let $x, y \in \mathbb{R}^n$ such that $\|x\| = 2$ and $\|y\| = 3$ and the angle between them is $\frac{\pi}{3}$. Evaluate $\|x - y\|$ (6marks)

QUESTION THREE (20 MARKS)

- a) Find the reduced row echelon form (r.r.e.f.) of the following matrix: (5marks)

$$\begin{bmatrix} 2 & 4 & 6 & 0 \\ 1 & 2 & 4 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix}$$

- b) Let A be a non-singular 4×4 matrix with $|A^{-1}| = 3$. Find

- i. $|\text{adj}(A)|$ (4marks)

- ii. $|\frac{1}{2}A^T \text{Adj}(A^{-1})|$ (6marks)
- c) Find all vectors in \mathbb{R}^4 which are perpendicular to the vectors $X = (1, 1, 2, 2)$ and $Y = (2, 3, 5, 5)$. (5marks)

QUESTION FOUR (20 MARKS)

- a) Let u and v be two solutions of the non-homogenous system $Ax = B$. Show that $u - v$ is a solution to the homogenous system $Ax = 0$. (4marks)
- b) Verify that the triangle with vertices $A(1, 1, 2)$, $B(1, 2, 3)$, and $C(3, 0, 3)$ is a right triangle. (5marks)
- c) Solve the following linear system using Gauss-Jordan method (6marks)
- $$\begin{aligned} x + 3y - z + w &= 1 \\ 2x - y - 2z + 2w &= 2 \\ 3x + y - z + w &= 1 \end{aligned}$$
- d) Show that if x and y are in \mathbb{R}^n , then $\|x + y\| \leq \|x\| + \|y\|$ (5marks)

QUESTION FIVE (20 MARKS)

- a) Let u and v be two solutions of the homogenous system $Ax = 0$. Show that $ru + sv$ (for $r, s \in \mathbb{R}$) is a solution to the same system. (4marks)
- b) Show that $U \cdot (V + W) = U \cdot V + U \cdot W$, for any vectors $U, V, W \in \mathbb{R}^n$. (4marks)
- c) Let A be a square matrix. Show that if $A = 2A^T$, then $A = 0$. (4marks)
- d) Find all values of a for which $X = (a^2 - a, -3, -1)$ and $Y = (2, a - 1, 2a)$ are orthogonal. (5marks)
- e) Show that if C_1 and C_2 are solutions of the system $Ax = B$, then $4C_1 - 3C_2$ is also a solution of this system. (3marks)