(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(MATHEMATICS)

COURSE CODE: MAP 211/MAT 201

COURSE TITLE: LINEAR ALGEBRA I

DATE: 18/07/2022 **TIME: 8:00 AM - 10:00 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO (2) Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE

a) Use Gauss- Jordan elimination to solve the following system of equations

$$4x+8y-12z=44$$
$$3x+6y-8z=32$$
$$-2x-y=-7$$

(6 marks)

b) Let **u** = (1, 2, 3), **v** = (2,-3, 1) and **w** = (3, 2,-1)

i) Find the components of the vector **u**-3**u**+8**w** (2 marks)

ii) Find the scalars c_1, c_2, c_3 such that $c_1\mathbf{u}+c_2\mathbf{v}+c_3\mathbf{w} = (6, 14,-2)$ (6 marks)

b) Let **u** = (2,-1, 1), **v** = (1, 1, 2). Find< **u**, **v** > and the angle between these two vectors. (4 marks)

c) Show that T: $IR^2 \rightarrow IR^3$ defined by T(x, y) = (x, x+y, x-y) is a linear transformation. (2 marks)

d) Let $T: IR^3 \rightarrow IR^3$ be denoted by

$$T\begin{Bmatrix}1\\0\\0\end{Bmatrix} = \begin{Bmatrix}1\\1\\0\end{Bmatrix},\quad T\begin{Bmatrix}0\\1\\0\end{Bmatrix} = \begin{Bmatrix}2\\0\\1\end{Bmatrix},\quad T\begin{Bmatrix}0\\0\\1\end{Bmatrix} = \begin{Bmatrix}1\\0\\1\end{Bmatrix}$$

a) Find the matrix representation of T with respect to the standard basis S of IR^3 (2 marks)

b) Find $T\begin{matrix}1\\2\\3\end{matrix}$ using the definition of T and then the matrix obtained in (a) (8 marks)

QUESTION TWO

a) Show that IR^n = span $\{e_1, e_2, \text{-----} e_n\}$ where e_i is the vector with 1 in the i^{th} component and 0 otherwise. (5 marks)

b) Show that the vectors **w** = (9,2,7) is a linear combination of the vectors **u** = (1,2,-1) and **v** = (6,4,2) whereas the vector **w**′ = (4,-1,8) is not. (10 marks)

c) Show that **a** = (1,0,1,2), **b** = (0,1,1,2), and **c** = (1,1,1,3) are linearly independent. (5 marks)

QUESTION THREE

a) Given that **u** = (2,-1,3) and **w** = (4,-1,2), find

   i) $\mathbf{u}_1$, the projection of **u** onto **w** (5 marks)

   ii) $\mathbf{u}_2$, the perpendicular vector to **w** (3 marks)

b) Given that **u** = (2,-1, 1) and **v** = (1, 1,-1), show that **u** and **v** are orthogonal. (2 marks)

c) If **u** = (1,2,-2) and **v** = (3,0,1) find the cross product **u** x **v** (5 marks)

d) Let **u** = (1, 2,-2) and **v** = (3, 0, 1). Show that <**u, u** x **v**> and <**v, u** x **v**> = 0 and hence **u** x **v** is orthogonal to both **u** and **v**. (5 marks)

QUESTION FOUR

a) Given a vector **v** = (a, b, c,) in IR^3

   i) Show that $\cos \alpha = \frac{a}{\|\mathbf{v}\|}$ (2 marks)

   ii) Find cos β (2 marks)

   iii) Find cos γ (2 marks)

   iv) Show that $\frac{\mathbf{v}}{\|\mathbf{v}\|} = (\cos \alpha, \cos \beta, \cos \gamma)$ (2 marks)

   v) Show that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ (2 marks)

b) Let V be a vector space, **u**ϵV and α is a scalar. Prove that the following properties hold.

   i) $0\mathbf{u} = 0$ (2 marks)

   ii) $\alpha 0 = 0$ (2 marks)

   iii) $(-1)\mathbf{u} = -\mathbf{u}$ (2 marks)

   iv) If $\alpha\mathbf{u} = 0$ then $\alpha = 0$ or $\mathbf{u} = 0$ (4 marks)

QUESTION FIVE

a) What is a homogenous system of linear equations? (2 marks)

b) Distinguish between a matrix of coefficients and an augmented matrix (2 marks)

c) Solve the following system of linear equations (8 marks)

$x_1-2x_2+4x_3 = 12$

$2x_2-x_2+5x_3 = 18$

$-x_1+3x_2-3x_3 = -8$

d) Use Gauss Jordan elimination to solve the following (8 marks)

$x_1+x_2+x_3 = 2$

$2x_1+3x_2+x_3 = 3$

$x_1-x_2-2x_3 = -6$