



*(Knowledge for Development)*

**KIBABII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**  
**2021/2022 ACADEMIC YEAR**  
**FIRST YEAR SECOND SEMESTER**  
**SPECIAL EXAMINATION**  
**FOR THE DEGREE OF MASTER OF SCIENCE IN**  
**PURE MATHEMATICS**

**COURSE CODE: MAT 832**

**COURSE TITLE: RINGS AND MODULES**

**DATE: 18/07/22 TIME: 8 AM -11 AM**

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**INSTRUCTIONS TO CANDIDATES**

Answer Question ONE and Any TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages<sup>1</sup>. Please Turn Over.

### QUESTION ONE (20 MARKS)

- a. Define the following
- i. Commutative ring (1mark)
  - ii. Division ring (1 mark)
  - iii. Field (1 mark)
- b. Show that if  $R$  is a ring then
- i. The zero element is unique (1 mark)
  - ii. The negative of any element is unique (1mark)
  - iii. The unit is unique (1mark)
- c. Let  $R$  be a ring. Show that
- i.  $0a = a0 = 0$  (3marks)
  - ii.  $(-a)b = a(-b) = -ab$  (3 marks)
- d. Show that all ideals of  $\mathbb{Z}$  are of the form  $\mathbb{Z}_n$  for some  $n \in \mathbb{Z}$ . (8 marks)

### QUESTION TWO (20 MARKS)

- a. Define the following
- i. An ideal generated by  $F$  (1mark)
  - ii. An ideal (2 mark)
  - iii. Subring (2marks)
  - iv. Ring homomorphism (3marks)
- b. Let  $R$  be a ring and let  $(I_t)_{t \in T}$  be a collection of ideals in  $R$ . Show that  $\bigcap_{t \in T} I_t$  is an ideal in  $R$ . (7marks)
- c. Let  $p$  be a prime number and  $a$  and  $b$  be integers. Show that if  $p \mid ab$  then  $p \mid a$  and  $p \mid b$  (5marks)

### QUESTION THREE (20 MARKS)

- a. Define the following
- i. Kernel of  $\varphi$  (1mark)
  - ii. Image of  $\varphi$  (1mark)
- b. Let  $\varphi: R \rightarrow S$  be a ring homomorphism. Show that
- i.  $\ker \varphi \subset R$  is an ideal (4marks)
  - ii.  $\text{im} \varphi \subset S$  is a subring (4marks)
- c. Show that a ring homomorphism  $\varphi: R \rightarrow S$  is injective if and only if  $\ker \varphi = 0$  (5marks)

- d. Let  $R$  be a principal ideal domain and  $a, b \in R$ . Show that  $d = \gcd(a, b)$  if and only if (a, b) = (d).  
 (5marks)

**QUESTION FOUR (20 MARKS)**

- a. Define the following domain
- i. Integral (1mark) divisor
  - ii. Zero (1mark) (1mark )
  - iii. Invertible (1mark)
  - iv. Associates (1mark)
  - v. Principal ideal (3)
  - vi. Least common multiple marks)
- b. Let  $R$  be a principal ideal domain. Show that any irreducible element in  $R$  is prime(6 marks)
- c. Let  $R$  be an integral domain. Show that two elements  $a, b \in R$  are associates if and only if (a) = (b).  
 (6marks)

**QUESTION FIVE (20 MARKS)**

- a. Define the following (2marks)
- i. Submodule (2marks)
  - ii. Indecomposable module (6 marks)
- b. Let  $K$  be a field and  $p \in K[x]$  be irreducible. Show that  $K[x]/(p)$  is a field (6marks)
- c. Let  $L$  be a field. Show that an intersection of a collection of subfields of  $L$  is a field (6marks)
- d. Show that for any simple  $R$ -module  $M$ , the endomorphism ring is a division ring (4marks)