(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAA 413**

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATION II

DATE: 26/05/2022 **TIME**: 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION THREE (20 MARKS)

a) Use matrix method to solve the non-homogenous system of equation. (12 marks)

$$\frac{dx_1}{dt} = 4x_2 - 3x_1 + 2$$
$$\frac{dx_2}{dt} = 2x_1 - x_2 + 2 \quad \text{and } X(0) = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

b) Use Elimination method to solve the system. (8 Marks)

$$\frac{dx}{dt} + \frac{dy}{dt} - x = 2t + 1$$

$$2\frac{dx}{dt} + 2\frac{dy}{dt} + x = t$$

QUESTION FOUR (20 MARKS)

Find the power series solution for the initial value problem

$$(1 - x^2)y^{II} + 2y = 0$$

$$y(2) = 4 \text{ and } y^I(2) = 5$$

at the ordinary point $x = 2$. (20 Marks)

QUESTION FIVE (20 MARKS)

a) Use reduction of order method to solve the differential equation by

$$y^{II} - 2xy^I - 2y = 0$$

given that $y = e^{x^2}$ is a solution to the differential equation. (10 Marks)

b) Solve the differential equation defined by

$(x^2 - 1)y^{II} - 2xy^I + 2y = o$ given that $y = x$ is a solution of the differential equation. (10 marks)

QUESTION ONE (30 MARKS)

a) Define the following. (3 Marks)
   i) Bifurcation
   ii) Stability
   iii) Equilibrium point

b) Use Picard's method to approximate the value of y when x=0.1 given that y = 1 when x = 0 and $\frac{dy}{dx} = x + y$. (4 marks)

c) Evaluate $W[f_1, f_2]$. Given that $f_1(x) = x^2$ and $f_2(x) = x + Inx$. (5 marks)

d) Find the general solution of the system $X' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} x$ (6 marks)

e) Solve the system $\begin{aligned} \frac{dx}{dt} + y = S\text{int} \\ \frac{dy}{dt} + x = Cost \end{aligned}$. (6 marks)

f) A non-linear system is described as $X' = \begin{bmatrix} x_1^2 - x_2^2 - 1 \\ 2x_2 \end{bmatrix} x$. Find the critical points and check their stability. (6 marks)

QUESTION TWO (20 MARK

a) State the condition for the following critical points to occur and in each case draw the phase portrait
   i) Saddle point. (2 marks)
   ii) Anode . (2 marks)

b) Consider a system described by $\frac{dx}{dt} = \mu - x^2$. Analyse the system and state the type of bifurcation. (8 marsk)

c) Consider a system described by $\frac{dx}{dt} = \mu x - x^2$. Analyse the system and state the type of bifurcation. (8 marks)