(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

SUPPLEMENTARY/SPECIAL EXAMINATION

FOR THE DEGREE OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 100**

COURSE TITLE: **MATHEMATICS FOR TECHNOLOGIST**

DATE: 20/07/2022 **TIME:** **8:00 AM – 10:00 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following terms
   i. Population
   ii. Complement sets
   iii. Geometric progression (3mks)

b) Show that $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ has complex eigenvalues (3mks)

c) At Dan's automatic shop 50 cars were inspected. 23 Needed new brakes, 34 needed new exhaust systems and 6 needed neither repair.
   i. How many needed both repairs
   ii. How many needed new brakes but not new exhaust system. (6mks)

d) Evaluate $\lim_{x\to 1}\frac{x^2+x-2}{x^2-3x+2}$ (2mks)

e) Prove that $\lim_{x\to 3}(2x+1) = 7$ (3mks)

f) Find the equation of the tangent and normal to the curve $y = x^2 - 4x + 1$ at the point $(-1,1)$. (4mks)

g) Determine if the following function is continuous at $x = 3$

$$f(x) = \begin{cases} x^2 - 4 & x < 3 \\ 5 & x = 3 \\ x + 2 & x > 3 \end{cases}$$ (4 mks)

h) Find the slope of the line tangent to its graph at the point $(2,2)$ of the equation (3mks)

$$y^2 + x^2y = 3x^2$$

i) Find $\frac{d^2y}{dx^2}$ given that $y = e^{-3x}cos4x$ (2mks)

QUESTION TWO (20MARKS)

a) Proof the DeMorgan's theorems
   i. $(A \cup B)^1 = A^1 \cap B^1$
   ii. $(A \cap B)^1 = A^1 \cup B^1$ (10mks)

b) Find eigenvalues and eigenvectors of the matrix (10mks)

$$A = \begin{pmatrix} 1 & -4 & -2 \\ 0 & 3 & 1 \\ 1 & 2 & 4 \end{pmatrix}$$

QUESTION THREE (20MARKS)

a) Find from the 1st principles or using the delta method the derivative of

$f(x) = 4x^3 - x^2 + 3x + 6$ (8mks)

b) Find y^I if $x^3 + y^3 = 6xy$ hence find the tangent line at the point $(3,3)$ (5mks)

c) Differentiate $y = (x+1)^5(3x^4 - x + 3)^7$ (7mks)

QUESTION FOUR (20MARKS)

a) The third term geometric sequence is 3 and the fifth term is $\frac{3}{4}$. Write down the first four terms of the sequence. (6mks)

b) In an arithmetic progression, the thirteenth term is 27 and the seventh term is three times the second term. Find the first term, common difference and the sum of the first ten terms. (7mks)

c) A geometric progression has positive terms. The sum of the first six terms is nine times the sum of the first three terms. The seventh term is 320. Find the common ratio and the first term. Find the smallest value of n such that the sum to first n terms of the progression exceeds 10^6. (7mks)

QUESTION FIVE (20MARKS)

a) A survey of 300 workers yielded the following information: 231 belonged to a union, and 195 were Democrats. If 172 of the union members were Democrats, how many workers were in the following situations? (10mks)

i. belonged to a union or were Democrats
ii. belonged to a union but were not Democrats
iii. were Democrats but did not belong to a union
iv. neither belonged to a union nor were Democrats

b) In newly established university there are 38 lecturers and 3 lecture sections. Morning, afternoon and evening. 16 of the lecturers conduct their lectures in the morning, 18 in the afternoon and 24 in the evening. Because of insufficient staff 12 lecturers conduct their lectures both in the morning and evening, 9 both in the afternoon and evening while 10 both in the morning and afternoon. There are 3 lecturers whose work is supervision and don't conduct any lecture. Represent the information on a Venn diagram and hence find the number of lecturers who conduct lectures in; (10mks)

i. All sections.
ii. Only one session.
iii. At least two sections.
iv. At most two sections