(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: STA 211/STA 241

COURSE TITLE: STATISTICS AND PROBABILITY

DATE: FRI 20/07/2022 **TIME: 2:00 PM – 4:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION 1: (30 marks)

(a) Define the following terms as used in probability and statistics:

(i) A random variable (3 marks)

(ii) A probability function (3 marks)

(iii) A cumulative distribution function (3 marks)

(b) A process manufactures ball bearings whose diameters are normally distributed with mean 2.505 cm and standard deviation 0.008 cm. Specifications call for the diameter to be in the Interval 2.5 ± 0.01 cm. What proportion of the ball bearings will NOT meet the specifications? (9 marks)

(c)The lifetimes of batteries in a certain application are normally distributed with mean 50 hours and standard deviation 5 hours. Find the probability that a randomly chosen battery lasts,

(i) Between 42 and 52 hours (4marks)

(ii) More than 60 hours (4marks)

(iii) At most 55 hours (4marks)

QUESTION 2: (20 marks)

(a) For a Poisson distributed random variable, X with probability function,

$$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad \text{for } x=0, 1, 2, 3,\ldots\ldots\ldots$$

Show that the moment generating function for such a variable is given by

$$M_x(t) = e^{\lambda(e^t-1)}$$

(8 marks)

(b) Use the expression of $M_x(t)$ given in 2(a) above to work out;

(i) The expected value of X, E(X) (4 marks)

(ii) The variance of X, Var(X) (5 marks)

Comment on the results obtained in 2b (i) and 2b (ii) in the immediate above.(3marks)

QUESTION 3: (20 marks)

(a) If X has a binomial distribution with the parameters n and p, with probability function

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad \text{for } x=0,1,2,3,\ldots\ldots,n,$$

Derive the expressions for:

(i) E(X)
(ii) Var(X) (4 marks)

What happens to the probability distribution of X when n=1? (6 marks) (1 mark)

(b) Suppose in a given experiment each air sample has a 10% chance of containing a particular rare molecule. Assume the samples are independent with regard to the presen of the rare molecule. Find the probability that in the next 18 samples;
(i) Exactly two samples contain the rare molecule. (3marks)
(ii) At most two samples contain the rare molecule. (3marks)
(iii) Between 1 and 4 samples contain the rare molecule (3marks)

QUESTION 4: (20 marks)

(a) Consider a random variable X to be normally distributed with moment generating function, $M_x(t)$ given by

$$M_x(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$$

Show that;
(i) $E(X) = \mu$
(ii) $Var(X) = \sigma^2$ (6 marks) (8 marks)

(b) When $\mu = 0$ and $\sigma^2 = 1$, what would be the distribution of X? (1mark)

(c) Suppose that the amount of cosmic radiation to which a person is exposed when flying by jet across Kenya is a random variable having a normal distribution with a mean of 4.35 mrem and a standard deviation of 0.59 mrem. What is the probability that a person will be exposed to more than 5.20 mrem of cosmic radiation on such a flight?. (5 marks)

QUESTION 5: (20 marks)

(a) Let $\overline{X}$ and S^2 be the mean and the variance of a random sample of size n from a normal population with mean, μ and the variance, σ^2.
Prove that,

$$T = \frac{\overline{X} - \mu}{S/\sqrt{n}}$$

has the t distribution with (n-1) degrees of freedom (10 marks)

(b) In 16 one-hour test runs, the gasoline consumption of an engine averaged 16.4 gallons with a standard deviation of 2.1 gallons. Test the claim that the average gasoline consumption of this engine is 12.00 gallons per hour. (7 marks)

(c) If S_1^2 and S_2^2 are the variances of independent random samples of size n_1 and n_2 from normal populations with variances σ_1^2 and σ_2^2, write an expression for the F distributed random variable, F.

(3marks)