(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN STATISTICS

COURSE CODE: **STA 801**

COURSE TITLE: **MEASURE THEORY AND PROBABILITY**

DATE: 26/05/2021 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One any other Two Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

1. (a) State and explain a probability space using its standard notation. (5 mks)

   (b) Suppose $E_1, E_2 \in \mathcal{F}$, Show that $\mathcal{F}$, is an algebra. (5 mks)

   (c) Let $0 \leq f_n \to f$ almost everywhere and $\int f_n d\mu \leq A < \infty$, show that f is integrable and $\int f d\mu \leq A$ (5 mks)

   (d) Let X and Y be independent random variables. Show that

   $$E[X|Y = y] = E[X]$$

   (5 mks)

   (e) Suppose $(X, \mathcal{F}, \mu)$ is a measure space and f and g are measurable functions on X and $A, B \in \mathcal{F}$. State the properties of f and g. (5 mks)

   (f) If $X_1, X_2, \ldots$ are independent random variables such that $E[x_n] = \mu$ and $Var[X_n] \leq \sigma^2$ for each n, show that $\frac{X_1+\ldots+X_n}{n} \to \mu$ in probability. (5 mks)

QUESTION TWO (20 MARKS)

2. (a) What are Lebesgue measurable sets? (2 mks)

   (b) Describe any two Lebesgue measurable sets (4 mks)

   (c) State and explain any four measurable functions (8 mks)

   (d) Show that if $\{f_n\}$ is a sequence of non-negative measurable functions, and $\{f_n(x) : n \leq 1\}$ increases monotonically to $f(x)$ for each x then

$$\lim_{n \to \infty} \int_E f_n(x) d\mu = \int_E f d\mu$$

(6 mks)

QUESTION THREE (20 MARKS)

3. (a) Let $A_1, A_2, \ldots$ be a sequence of events. Show that:

      i. if $\sum P(A_n) < \infty$ then $P(\limsup A_n) = 0$ (5 mks)

      ii. if $\sum P(A_n) = \infty$ then $P(\limsup A_n) = 1$ (5 mks)

   (b) Suppose $f = \sum_i x_i I_{Ai}$ is a non negative simple function, $\{A_i\}$ being decomposition of S into F sets, show that

$$\int f d\mu = \sum_i x_i \mu(A_i)$$

(6 mks)

   (c) State five (equivalent) conditions for the random variables $X_1, \ldots, X_n$ to be independent. (4 mks)

QUESTION FOUR (20 MARKS)

4. (a) Suppose X is a random with the distribution μ_X. Show that $E(X) = \int\limits_R x \ d\mu_X$ (10 mks)

   (b) Suppose $\{B_n\}$ is sequence of independent events and $\sum\limits_n P\{B_n\} = \infty$. Show the probability that B_n occurs infinitely often is one. (10 mks)

QUESTION FIVE (20 MARKS)

5. (a) State and explain two properties of conditional expectation (4 mks)

   (b) Find the mathematical expectation of a random variable with: (6 mks)

   (c) Let $f_n \geq 0$ be a measurable function. Show that $\int\limits_x \liminf f_n d\mu \leq \liminf \int\limits_x f_n d\mu$ as $n \to \infty$ (10 mks)