(Knowledge for Development)

KIBABII UNIVERSITY

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **STA 412**

COURSE TITLE: PROBABILITY AND MEASURE

DATE: 25/05/2022 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

1. (a) Define the following terms

    i. Probability space (1 mk)

    ii. Borel set (1 mk)

    iii. Sigma-algebra (1 mk)

    iv. Measurable sets (1 mk)

   (b) Describe any two Lebesgue measurable sets. (4 mks)

   (c) Let $A, B \subset E$ such that $\mu^*(A)$ and $\mu^*(B)$ are both finite. Show that, $|\mu^*(A) - \mu^*(B)| \leq \mu^*(A\Delta B)$ where $(A\Delta B) := (AB^c) \cup (BA^c)$ (5 mks)

   (d) If $A \subset B$, show that $\mu^*(A) \geq \mu^*(B)$. (3 mks)

   (e) Prove that if $0 \leq f_n \to f$ almost everywhere and $\int f_n d\mu \leq A < \infty$, then f is integrable and $\int f d\mu \leq A$ (3 mks)

   (f) State and explain any two types of measures on the intervals over the real line. (5 mks)

   (g) Suppose that $A, B \in \mathcal{A}$. Show that $\mu(B) = \mu(A \cap B) + \mu(B \cap A')$ (3 mks)

   (h) Let $\{F_i \subset R^n : i \in N\}$ is countable collection of R^n. Show that

$$\sum_{i=1}^{\infty} \mu^*(F_i) \geq \mu^*(\bigcup_{i}^{\infty} F_i)$$

(3 mks)

QUESTION FOUR (20 MARKS)

4. (a) State and explain two properties of conditional expectation (4 mks)

   (b) Find the mathematical expectation of a random variable with (9 mks)

      i. uniform distribution over the interval $[a, b]$

      ii. triangle distribution

      iii. exponential distribution

   (c) Show that if $\{f_n\}$ is a sequence of positive measurable functions, and $\{f_n(x) : n \leq 1\}$ increases monotonically to $f(x)$ for each x then

$$\lim_{n \to \infty} \int_E f_n(x)dm = \int_E f dm$$

(7 mks)

QUESTION FIVE (20 MARKS)

5. (a) What are Lebesgue measurable sets? (2 mks)

   (b) Describe any two Lebesgue measurable sets (4 mks)

   (c) State and explain any four measurable functions (8 mks)

   (d) Show that if $\{f_n\}$ is a sequence of non-negative measurable functions, and $\{f_n(x) : n \leq 1\}$ increases monotonically to $f(x)$ for each x then

$$\lim_{n \to \infty} \int_E f_n(x)dm = \int_E f dm$$

(6 mks)