(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/SUPPLIMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **STA 221/242**

COURSE TITLE: PROBABILITY AND DISTRIBUTION MODELS

DATE: 26/07/2022 **TIME**: 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

(a) Define the term moment generating function (1 mk)

(b) State the relationship between a CDF and PDF as used in probability theory (1 mk)

(c) Given a probability distribution function

$$f(x) = \begin{cases} Cx^2 & ; o < x < 1 \\ 0 & elsewhere \end{cases}$$

Find the value of C and hence the median and interquartile range (6 mks)

(d) For a chi-square distribution with r degree of freedom and its distribution density given as;

$$f(x) = \begin{cases} \frac{1}{2^{r/2}\Gamma(r/2)} \cdot x^{\frac{r-2}{2}} \cdot e^{-\frac{x}{2}}, & x > 0 \\ 0, & elsewhere \end{cases}$$

Show that moment generating function,
$m(t) = (1-2t)^{-r/2}$, hence or otherwise find E(x) and Var(x). (10 marks)

(e) Let X and Y be two random variables each taking three values – 1, 0 and 1, and having the joint probability distribution

			x		
		- 1	0	1	Sum
y	- 1	0	0.1	0.1	0.2
	0	0.2	0.2	0.2	0.6
	1	0	0.1	0.1 .	0.2
	Sum	0.2	0.4	0.4	1.0

(i) Show that X and Y have different Expectations (2 mks)

(ii) Prove that X and Y are uncorrelated (3 mks)

(iii) Given that $Y = 0$, what is the conditional probability distribution of X(3 mks)

(iv) Find Var(Y| X = - 1) (4mks)

QUESTION TWO (20 MARKS)

Three coins are tossed. X denotes the number of heads on the first two coins, Y denotes the number of tails on the last two coins and Z denotes the number of heads on the last two coins. Required, find;

(a) The joint distribution of (i) X and Y (ii) X and Z

(b) The conditional distribution of Y given X = 1

(c) E(Z|X = 1)

(d) The correlation coefficient between X and Y

QUESTION THREE (20 MARKS)

(a). For a gamma distribution with parameters α andβ;

$$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1}e^{-y}dy \quad with \quad y = \frac{x}{\beta} \quad where \quad \beta > 0$$

Show that moment generating function is

$$m(t) = \frac{1}{(1-\beta \mathrm{t})^{t}}$$

Hence or otherwise using the moment generating function, find the Var(x). (12 marks)

(b) Given that $X \sim Exp(\beta)$ i.e. $f(x,\beta) = \frac{1}{\beta} e^{-x/\beta}$, x >0. Find the moment generating function of the distribution and hence it's mean and variance. (8 marks)

QUESTION FOUR (20 MARKS)

(a) Two random variables X and Y have the following joint probability density function

$$f(x,y) = \begin{cases} 2 - x - y & ; 0 \leq x \leq 1, \quad 0 \leq y \leq 1 \\ 0 & otherwise \end{cases}$$

Find

(i) Marginal probability density functions of X and Y (3mks)

(ii) Conditional density functions (3mks)

(iii)Covariance between X and y (5 mks)

(b). Consider the following bivariate function defined by;

$$f(x,y) = \begin{cases} k(6-\mathrm{x}-\mathrm{y}) & 0 < x < 2, 2 < y < 4 \\ 0, & otherwise. \end{cases}$$

Determine the value of the constant k such that $f(x,y)$ is the probability density function, hence evaluate the following.

(i). $p(\mathrm{x} \leq 1, \quad \mathrm{y} \leq 3)$

(ii). $\mathrm{p}(\mathrm{x}+\mathrm{y} < 3)$ (9 mks)

QUESTION FIVE (20 MARKS)

The joint probability function of two discrete random variables X and Y is given by $f(x,y) = k(2x+y)$, where x and y assume all integers such that $0 \leq x \leq 2$, $0 \leq y \leq 3$, and $f(x,y) = 0$ otherwise. The probabilities associated with these points, given by $k(2x+y)$, are shown in the table below;

				Y		
		0	1	2	3	Total
	0	0	k	$2k$	$3k$	$6k$
X	1	$2k$	$3k$	$4k$	$5k$	$14k$
	2	$4k$	$5k$	$6k$	$7k$	$22k$
	Total	$6k$	$9k$	$12k$	$15k$	$42k$

Required, find;

(i) the value of the constant k. (2mks)

(ii) $P(X=2, Y=1)$. (1mk)

(iii) $P(X \geq 1, Y \leq 2)$. (1mks)

(iv) the marginal probability function of X and Y. (4mks)

(v) Evaluate $E(X)$, $E(Y)$, $E(X,Y)$, $E(X^2)$, $E(Y^2)$, $Var(X)$, $Var(Y)$ and $Cov(X,Y)$ (9mks)

(vi) Show that the random variables X and Y are independent. (3mks)

END