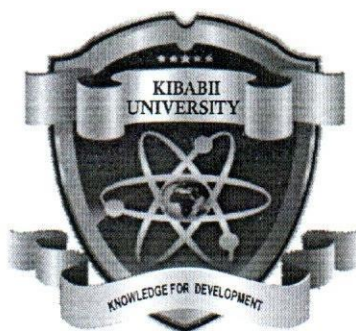


15



(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/SUPPLIMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: STA 221/242

COURSE TITLE: PROBABILITY AND DISTRIBUTION MODELS

DATE: 26/07/2022

TIME: 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

- (a) Define the term moment generating function (1 mk)
 (b) State the relationship between a CDF and PDF as used in probability theory (1 mk)
 (c) Given a probability distribution function

$$f(x) = \begin{cases} Cx^2 & ; 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the value of C and hence the median and interquartile range (6 mks)

- (d) For a chi-square distribution with r degree of freedom and its distribution density given as;

$$f(x) = \begin{cases} \frac{1}{2^{r/2} \Gamma(r/2)} \cdot x^{\frac{r-2}{2}} \cdot e^{-\frac{x}{2}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Show that moment generating function, $m(t) = (1 - 2t)^{-r/2}$, hence or otherwise find $E(x)$ and $Var(x)$. (10 marks)

- (e) Let X and Y be two random variables each taking three values $-1, 0$ and 1 , and having the joint probability distribution

		x			Sum
		- 1	0	1	
y	- 1	0	0.1	0.1	0.2
	0	0.2	0.2	0.2	0.6
	1	0	0.1	0.1	0.2
Sum		0.2	0.4	0.4	1.0

- (i) Show that X and Y have different Expectations (2 mks)
 (ii) Prove that X and Y are uncorrelated (3 mks)
 (iii) Given that $Y = 0$, what is the conditional probability distribution of X (3 mks)
 (iv) Find $Var(Y | X = -1)$ (4mks)

QUESTION TWO (20 MARKS)

Three coins are tossed. X denotes the number of heads on the first two coins, Y denotes the number of tails on the last two coins and Z denotes the number of heads on the last two coins. Required, find;

- (a) The joint distribution of (i) X and Y (ii) X and Z
 (b) The conditional distribution of Y given $X = 1$
 (c) $E(Z|X = 1)$
 (d) The correlation coefficient between X and Y

QUESTION THREE (20 MARKS)

- (a). For a gamma distribution with parameters α and β ;

$$\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1} e^{-y} dy \quad \text{with } y = \frac{x}{\beta} \quad \text{where } \beta > 0$$

Show that moment generating function is

$$m(t) = \frac{1}{(1 - \beta t)^t}$$

Hence or otherwise using the moment generating function, find the $\text{Var}(x)$. (12 marks)

(b) Given that $X \sim \text{Exp}(\beta)$ i.e. $f(x, \beta) = \frac{1}{\beta} e^{-x/\beta}$, $x > 0$. Find the moment generating function of the distribution and hence its mean and variance. (8 marks)

QUESTION FOUR (20 MARKS)

(a) Two random variables X and Y have the following joint probability density function

$$f(x, y) = \begin{cases} 2 - x - y & ; 0 \leq x \leq 1, \quad 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find

- (i) Marginal probability density functions of X and Y (3mks)
- (ii) Conditional density functions (3mks)
- (iii) Covariance between X and y (5 mks)

(b). Consider the following bivariate function defined by;

$$f(x, y) = \begin{cases} k(6 - x - y) & 0 < x < 2, 2 < y < 4 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the value of the constant k such that $f(x, y)$ is the probability density function, hence evaluate the following.

- (i). $p(x \leq 1, y \leq 3)$
- (ii). $p(x + y < 3)$ (9 mks)

QUESTION FIVE (20 MARKS)

The joint probability function of two discrete random variables X and Y is given by $f(x, y) = k(2x + y)$, where x and y assume all integers such that $0 \leq x \leq 2$, $0 \leq y \leq 3$, and $f(x, y) = 0$ otherwise. The probabilities associated with these points, given by $k(2x + y)$, are shown in the table below;

		Y				Total
		0	1	2	3	
X	0	0	k	$2k$	$3k$	$6k$
	1	$2k$	$3k$	$4k$	$5k$	$14k$
	2	$4k$	$5k$	$6k$	$7k$	$22k$
	Total	$6k$	$9k$	$12k$	$15k$	$42k$

Required, find;

- (i) the value of the constant k .

(2mks)

- (ii) $P(X = 2, Y = 1)$. (1mk)
- (iii) $P(X \geq 1, Y \leq 2)$. (1mks)
- (iv) the marginal probability function of X and Y . (4mks)
- (v) Evaluate $E(X)$, $E(Y)$, $E(X, Y)$, $E(X^2)$, $E(Y^2)$, $Var(X)$, $Var(Y)$ and $Cov(X, Y)$ (9mks)
- (vi) Show that the random variables X and Y are independent. (3mks)

END