(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

COURSE CODE: **MAT 823**

COURSE TITLE: **FUNCTIONAL ANALYSIS II**

DATE: 26/05/2021 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 2 Hours

QUESTION ONE (20 MARKS)

a) Define the following

  i. Parallelogram equality

  ii. Orthogonal

b) Show that space $\mathcal{L}^2$ is a Hilbert space is a Hilbert space with inner product defined by $< x, y >= \sum_{j=1}^{\infty} \varepsilon_j \eta_j$

c) Show that the space $\mathcal{L}^p$ $(p \neq z)$ is not an inner product space, hence not a Hilbert Space

d) Show that for any inner product space X, there exists a Hilbert space H and an isomorphism A from X on to a dense subspace $W \subseteq H$ such that the space H is unique except for isomorphisms.

QUESTION TWO (20 MARKS)

a) Define the following terms

  i. Segment

  ii. Convex

b) Given M is a complete subspace of Y and $x \in X$ fixed, show that $z = x - y$ is orthogonal to Y

c) Show that an orthogonal set is linearly independent

d) State the Bessel inequality

QUESTION THREE (20 MARKS)

a) Define the following terms

  i. Total orthonormal set

  ii. Parseval relation

  iii. Isomorphism

b) State Riesz's theorem

c) Let H_1, H_2 be Hilbert spaces and $h: H_1 \times H_2 \to K$ a bounded sesquilinear form. Show that h has a representation $h(x, y) =< sx, y >$ where $S: H_1 \to H_2$ is a bounded linear operator. S is uniquely determined by h and has norm $\parallel s \parallel = \parallel h \parallel$

QUESTION FOUR (20 MARKS)

a) Define the adjoint operator $T^{\times}$

b) Show that the adjoint operator $T^{\times}$ is linear and bounded and $\| T^{\times} \|=\| T \|$

c) Show that every Hilbert space H is reflexive

d) State the uniform Boundedness Theorem

QUESTION FIVE (20 MARKS)

a) Define the following terms

  i. Strong convergence

  ii. Weak convergence

  iii. Uniformly operator convergent

  iv. Strongly operator convergent

  v. Weakly operator convergent

b) Show that an A-summability method with matrix $A = (\propto_{nk})$ is regular if and only if

  i. $\lim_{n\to\infty} \propto_{nk} = 0$ for k=1,2….

  ii. $\lim_{n\to\infty} \sum_{k=1}^{\infty} \propto_{nk} = 0$

  iii. $\sum_{k=1}^{\infty} | \propto_{nk}| \leq r$ for n=1,2 where r is a constant which does not depend on n

c) State the open mapping theorem