(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION (SB)

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: MAT 422/MAA 421

COURSE TITLE: PDE II

DATE: 7/9/2022 **TIME:** 2:00PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE COMPULSORY (30 MARKS)

(a) Define the following terms;

i. Reducible linear differential operator.

ii. Linear non-homogeneous partial differential equation. (4 marks)

(b) Solve $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = x$ (6 marks)

(c) Determine the complete solution of

$$\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x + 2y)$$ (6 marks)

(d) Using method of separation of variables, solve

$\frac{\partial u}{\partial x} = 3\frac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-5x}$ (6 marks)

(e) Using method of characteristics solve the semi-linear partial differential equation.

$4\frac{\partial u}{\partial y} - 2\frac{\partial u}{\partial x} + 5u = 0$ (4 marks)

(f) Given the following PDE;

$$y\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial y^2} = 0$$

i. Determine the region in the $xy - plane$ where it is elliptic.

ii. Determine its characteristic curves. (4 marks)

QUESTION TWO (20 MARKS)

(a) Differentiate between Laplace's equation and Poisson's equation. (2 marks)

(b) Find the temperature function $u(x,t)$ on an insulated metallic rod of length L which is governed by $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ under the conditions $u(0,t) = 0, u(L,t) = 0$and $u(L,t) = \frac{100\,x}{L}$ (13 marks)

(c) Solve $(D^2 + D')(D + 4D' - 6)z = 0$ (5 marks)

QUESTION THREE (20 MARKS)

(a) Classify the given PDE below and determine its characteristic curves

$$y^2\frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x \partial y} + x^2\frac{\partial^2 u}{\partial y^2} = y^3\frac{\partial u}{\partial x} + x^4\frac{\partial u}{\partial y}$$ (3 marks)

(b) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} - 2z = x^2y^2$ (11 marks)

(c) Solve the wave equation by D'Alembert's method

$\frac{\partial^2 y}{\partial t^2} = C^2\frac{\partial^2 y}{\partial x^2}$ where C is a constant (6 marks)

QUESTION FOUR (20 MARKS)

Given the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial^2 u}{\partial y^2} + 1 = 0 \text{ in } 0 \le x \le 1, y > 0 \text{ with } u = \frac{\partial u}{\partial y} = x \text{ on } y = 0$$

i. Classify the PDE (2 marks)

ii. Reduce it to canonical form and solve it (18 marks)

QUESTION FIVE (20 MARKS)

(a) Solve the following wave equation using the method of separation of variables

$$\frac{\partial^2 y}{\partial t^2} = 25\frac{\partial^2 y}{\partial x^2}$$

with boundary conditions

$y = 0$, when $x = 0$ and $x = 2$

and initial conditions

$\frac{\partial y}{\partial t} = 0$, and $y(x, 0) = 6\sin\left(\frac{\pi x}{2}\right) - 3\sin(\pi x)$ when $t = 0$, $0 < x < 2$ (14 marks)

(b) Determine the complete solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} - 3\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = e^{2x+3y}$ (6 marks)